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Distributed Rendezvous Control of Networked Uncertain Robotic Systems with Bearing Measurements

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Abstract— **In this paper, the distributed rendezvous control problem of networked uncertain robotic systems with bearing measurements is investigated. The network topology of the multi-robot systems is described by an undirected graph. The dynamics of robots is modeled by Euler-Lagrange equation with unknown inertial parameters, which is more general than simple kinematics considered in existing works on rendezvous problem of multi-robot systems. To achieve rendezvous, a distributed adaptive force/torque control law is developed for each robot, which uses bearings with respect to its neighbors instead of relative displacements or distances. It is shown that the resulting closed-loop multi-robot systems are globally asymptotically stable. Then, the rendezvous control problem of multiple wheeled mobile robots is further solved by the proposed approach. Finally, on-site experiment on networked TurtleBot3 Burger mobile robots is conducted and the results demonstrate effectiveness of the proposed approach.**

Index Terms—**Multi-robot systems, motion control, networked robots**

I. INTRODUCTION

Rendezvous problem is one of the basic tasks in control of multi-robot system, which has attracted much research attention in recent years; see, for example [1]–[8]. Specifically, for the kinematic models of single- or double-integrators with relative displacement measurements, rendezvous problem is the same as the consensus problem [9]. Since the mathematical models of most real robots are described by nonlinear and nonholonomic systems, many works focused on the rendezvous problem of unicycle-type mobile robots; see [1], [3], [4], [6], in which the orientations of robots are taken into consideration. However, the dynamics of most robotic systems are more complicated, and their kinematics are dependent on the dynamics. Therefore, it is of much significance to investigate rendezvous problem for robots with more general dynamics.

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The mathematical model of robots such as manipulators, wheeled mobile robots, drones, underwater vehicles, and walking robots [10]–[12], are usually described by Euler-Lagrange equation. Based on this model, many remarkable works on motion control of multiple robotic systems have been shown, including consensus [13], [14], trajectory tracking [15], distributed optimization [16], containment control [17], formation tracking control [18], as well as rendezvous problem [8]. In particular, adaptive control approaches were proposed in [8], [13] based on relative displacement measurements between neighboring robots, such that the leader-following consensus and rendezvous problems were solved respectively. However, it may not be easy for multi-robot systems to directly and accurately measure the real-time inter-robot distances in some scenarios, especially for robots with merely visual sensors such as cameras.

Recently, many works have devoted to control of multi-robot system with bearing measurements, including the rendezvous problem [4]–[6], the target-enclosing problem [19], and the formation control problem [6], [20]–[24], since it is easier to acquire the bearing measurements than the relative-displacement or relative-distance ones by robots' onboard visual sensors. Particularly, the rendezvous problems of kinematic multi-robot systems with bearing measurements were solved in [4]–[6]. However, to the best of our knowledge, the rendezvous control problem of dynamic multi-robot systems with bearing measurements still remains open.

This paper aims to solve the distributed rendezvous control problem of networked uncertain robots with bearing measurements. The uncertainty lies in the unknown inertial parameters including mass and moment of inertia. The topology of sensing network is described by a connected undirected graph. First, an adaptive force/torque control law is proposed for each robot based merely on the bearing measurements with respect to its neighbors and its own position and velocity. The global asymptotic stability of the resulting closed-loop system is established. Then, we apply the obtained result to rendezvous control of multiple wheeled mobile robots. Finally, experiment on networked TurtleBot3 Burger mobile robots are shown to illustrate the effectiveness of the proposed approach.

The main contributions are summarized as follows. First, the proposed control law solves the rendezvous problem of uncertain dynamic robotic systems which are more general and fundamental than those in most existing works where kinematic models were considered [1]–[7]. Second, compared with [8], [13] which considered leader-following consensus and rendezvous problem of uncertain dynamic robotic systems, our proposed control law only requires each robot to measure bearings instead of relative displacements with respective to its neighboring robots. Moreover, our work considers the leadless rendezvous problem which cannot be handled in those leader-following frameworks, for instance the one in [8]. Last but not least, unlike just simulation results were provided in most existing works on the multi-robot system control, this paper includes experimental implementation on TurtleBot3 Burger mobile robots and the proposed bearing-based rendezvous control of real robots is first realized. The results of on-site experiment convincingly demonstrate the effectiveness.

The remainder of this paper is organized as follows. In Section II, the problem formulation is given. In Section III, the main results including control law design and stability analysis are presented. In Section IV, a case study on rendezvous of wheeled mobile robots is given. In Section V, on-site experiment are presented. Finally, the conclusion is drawn in Section VI.

II. PROBLEM FORMULATION

Consider N robots in $\mathbb{R}^d,\ d \geq 2$, of which dynamics are modeled as the following uncertain robotic systems,

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i,\ i = 1,\ldots,N, \quad (1)$$

where $q_i \in \mathbb{R}^d$, $\dot{q}_i \in \mathbb{R}^d$ are the generalized position and velocity of robot i, $M_i(q_i) \in \mathbb{R}^{d\times d}$ is the inertia matrix and *unknown* to robot i, $C_i(q_i,\dot{q}_i) \in \mathbb{R}^{d\times d}$ is the Coriolis and centrifugal matrix, $G_i(q_i) \in \mathbb{R}^d$ is the gravity vector, and τ_i is the generalized force as well as the control input to be designed based on the information of neighboring agents. It is well known that system (1) has the following properties.

Property 1: $M_i(q_i)$ is uniformly positive definite.

Property 2: $\dot{M}_i(q_i)$ and $C_i(q_i,\dot{q}_i)$ satisfy $x^{\mathrm{T}}(\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i))x = 0$ for all $x \in \mathbb{R}^d$

Property 3: For all $x, y \in \mathbb{R}^d$, it holds that $M_i(q_i)x + C_i(q_i,\dot{q}_i)y + G_i(q_i) = Y_i(q_i,\dot{q}_i,x,y)\theta_i$, where $Y_i(q_i,\dot{q}_i,x,y)$ is a known regression matrix and θ_i is a constant vector consisting of systems' physical parameters.

The network topology of the multi-robot systems is described as an undirected graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$, where node set $\mathcal{V} = \{1,\ldots,N\}$ denotes N robots and edge set $\mathcal{E} = \{(j,i) : j \neq i,\ i,j \in \mathcal{V}\}$ represents the sensing channel between robots i and j. As a fundamental assumption on control of multi-robot systems, the following assumption is required.

Assumption 1: Graph $\mathcal{G}$ is fixed and connected.

Moreover, a set including the neighbors of robot i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. For each $(i,j) \in \mathcal{E}$, define the bearing between robots i and j as

$$g_{ij} = \begin{cases} \frac{q_j - q_i}{\|q_j - q_i\|}, & q_j \neq q_i, \\ 0, & q_j = q_i. \end{cases} \quad (2)$$

Now, the distributed rendezvous control problem with bearing measurements is formulated as follows.

Problem 1: Consider N robots with the uncertain dynamics (1) and a network topology $\mathcal{G}$. For each robot $i \in \mathcal{V}$, with any initial position $q_i(t_0) \in \mathbb{R}^d$ and initial velocity $\dot{q}_i(t_0) \in \mathbb{R}^d$, design a dynamic control law in the form of

$$\begin{aligned} \dot{\rho}_i &= \varrho(q_i,\dot{q}_i,g_{ij},\dot{g}_{ij}), \\ \tau_i &= \sigma(\rho_i,q_i,\dot{q}_i,g_{ij},\dot{g}_{ij}), i \in \mathcal{V}, j \in \mathcal{N}_i, \end{aligned} \quad (3)$$

such that the relative displacements $q_i(t) - q_j(t),\ \forall i,j \in \mathcal{V}$, converge to zero as $t \to \infty$, where $\varrho(\cdot)$ and $\sigma(\cdot)$ are two sufficiently smooth functions to be designed.

Remark 1: It is obvious in Problem 1 that neither the position of neighbors' positions q_j nor the relative displacements $q_j - q_i$ with respect to neighbors is required, while the relative displacement $q_j - q_i$ is widely utilized in control law design, for example, those in [1], [8], [13]. Moreover, for each robot i, the physical constant vector θ_i is also unknown. In other words, robots are not required to know their own physical parameters including mess, moment of inertia or sizes.

Remark 2: Note that q_i and $\dot{q}_i$ are also required in most distributed control of robotic systems, for example [8], [15], [25]. In this paper, q_i and $\dot{q}_i$ are used to compute the real-time value of regression matrix Y_i subject to the unknown parameter θ_i. In fact, the knowledge of q_i and $\dot{q}_i$ may not be necessary in some cases, for instance, the case presented in the simulation example of [15] and the case with mobile wheeled robots given in Section IV.

Remark 3: System (1) can be used to describe the dynamics of many robotic systems including manipulator, wheeled mobile robots, drones, and underwater vehicles [10]. Naturally, the solution to Problem 1 can be applied to achieve rendezvous of heterogenous robots, which means that all robots finally share the same position/orientation. Note that the setup of Problem 1 does not involve collision avoidance or connectivity preservation which is our future interest. If robots i and j share the same position, we assign $g_{ij} = 0$ in (2). In fact, since robots has their own sizes, the bearing vectors can be well defined in practice. Finally, Assumption 1 is widely made in existing works on multi-robots systems including the rendezvous problem [3], [4], [8].

III. RENDEZVOUS CONTROL WITH BEARING MEASUREMENTS

In this section, a bearing-based adaptive force/torque control law is proposed to solve the rendezvous problem for networked uncertain robotic systems, i.e., Problem 1.

A. Bearing-Based Adaptive Force/Torque Control Law

Since the inertial matrix $M_i(q_i)$ is unknown to each robot $i \in \mathcal{V}$, we need to design an adaptive law such that the unknown physical parameters involved in $M_i(q_i)$ can be estimated. To estimate θ_i, we define $\hat{\theta}_i \in \mathbb{R}^d$ for each robot i. The force/torque control law is proposed as

$$\begin{aligned} \tau_i &= -k_c s_i + \dot{q}_{ri} + Y_i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri})\hat{\theta}_i, \\ \dot{\hat{\theta}}_i &= -\gamma_i Y_i^{\mathrm{T}}(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri})s_i, \end{aligned} \quad (4)$$

with the auxiliary variables

$$\dot{q}_{ri} = \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij},\ s_i = \dot{q}_i - \dot{q}_{ri}, \quad (5)$$

where k_c and γ_i are positive constants, and $a_{ij} > 0$ if $i \in \mathcal{V}, j \in \mathcal{N}_i$; otherwise $a_{ij} = 0$. The initial state of $\hat{\theta}_i$ can be arbitrarily selected in $\mathbb{R}^d$.

Since control law (4) depends only on the local measurement and require no communication between robots, implementation on the networked robotic systems can be fully distributed. To implement the control law in a distributed manner, robots are required to be equipped with an optical camera, as mentioned in [23], [24]. Accordingly, like in [23], [24], g_{ij} is directly measured and $\dot{g}_{ij}$ can be obtained based on the pin-hole camera. The real-time velocity $\dot{q}_i$ can be measured by a variety of velocity transducers such as DC or AC tachometers. The force/torque control law can be implemented on each robot by its equipped servomotors.

Now we present the main result of this paper as follows.

Theorem 1: Under Assumption 1, control law (4) solves Problem 1, and makes estimation errors $\hat{\theta}_i - \theta_i$, $\forall i \in \mathcal{V}$ bounded and convergent.

B. Stability Analysis

The proof of Theorem 1 is stated as follows.

Proof. Define the estimation error as $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $\forall i \in \mathcal{V}$. Then, substituting (4) into (1) leads to the closed-loop system

$$\ddot{q}_i = M_i^{-1}\left(-k_c s_i + \dot{q}_{ri} + (\tilde{\theta}_i + \theta_i)Y_i - (C_i\dot{q}_i + G_i)\right),$$
$$\dot{\tilde{\theta}}_i = -\gamma_i Y_i^{\mathrm{T}} s_i,\ i \in \mathcal{V}. \tag{6}$$

Consider a Lyapunov function candidate as $V = \sum_{i=1}^{N} V_i$ with

$$V_i = \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\|q_i - q_j\| + \frac{1}{2}s_i^{\mathrm{T}} M_i(q_i)s_i + \frac{1}{2}\gamma_i^{-1}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i, \tag{7}$$

which is positive definite for $t \geq t_0$ based on Property 1. Taking the upper right-hand time derivative of V along trajectories of systems (6) yields

$$\begin{aligned}
D^+V =& \sum_{i=1}^{N}\Big(\sum_{j\in\mathcal{N}_i} a_{ij}\frac{(q_i - q_j)^{\mathrm{T}}}{\|q_i - q_j\|}(\dot{q}_i - \dot{q}_j) \\
&+ s_i^{\mathrm{T}} M_i(q_i)(\ddot{q}_i - \ddot{q}_{ri}) + \frac{1}{2}s_i^{\mathrm{T}}\dot{M}_i(q_i)s_i + \tilde{\theta}_i^{\mathrm{T}}\gamma_i^{-1}\dot{\tilde{\theta}}_i\Big) \\
=& \sum_{i=1}^{N}\Big(-\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}^{\mathrm{T}}\dot{q}_i - k_c s_i^{\mathrm{T}} s_i + \sum_{j\in\mathcal{N}_i} a_{ij}s_i^{\mathrm{T}} g_{ij} \\
&+ s_i^{\mathrm{T}}(\tilde{\theta}_i + \theta_i)Y_i - s_i^{\mathrm{T}} C_i\dot{q}_i - s_i^{\mathrm{T}} G_i - s_i^{\mathrm{T}} M_i(q_i)\ddot{q}_{ri} \\
&+ \frac{1}{2}s_i^{\mathrm{T}}\dot{M}_i(q_i)s_i - \tilde{\theta}_i^{\mathrm{T}} Y_i^{\mathrm{T}} s_i\Big) \\
=& \sum_{i=1}^{N}\Big(-k_c\sum_{j\in\mathcal{N}_i} s_i^{\mathrm{T}} s_i - \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}^{\mathrm{T}}\dot{q}_i \\
&+ a_{ij}g_{ij}^{\mathrm{T}}(\dot{q}_i - \dot{q}_{ri}) + s_i^{\mathrm{T}}(\tilde{\theta}_i + \theta_i)Y_i - s_i^{\mathrm{T}} G_i \\
&- s_i^{\mathrm{T}} C_i(s_i + \dot{q}_{ri}) - s_i^{\mathrm{T}} M_i\ddot{q}_{ri} + \frac{1}{2}s_i^{\mathrm{T}}\dot{M}_i s_i - \tilde{\theta}_i^{\mathrm{T}} Y_i^{\mathrm{T}} s_i\Big) \\
=& \sum_{i=1}^{N}\Big(-k_c s_i^{\mathrm{T}} s_i - \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij}^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} + s_i^{\mathrm{T}}(Y_i\theta_i \\
&- M_i\ddot{q}_{ri} - C_i\dot{q}_{ri} - G_i) + \frac{1}{2}s_i^{\mathrm{T}}(M_i - 2C_i)s_i\Big) \\
=& -\sum_{i=1}^{N}((k_c s_i^{\mathrm{T}} s_i + q_{ri}^{\mathrm{T}} q_{ri}) \leq 0,
\end{aligned} \tag{8}$$

where the last equality holds based on Properties 2 and 3. Thus, system (6) is globally stable, which implies that estimation errors $\tilde{\theta}_i$, $\forall i \in \mathcal{V}$, bounded.

By the non-smooth LaSalle Invariance Principle [26, Theorem 3.2, Chapter VII], the trajectories of system (6) converge to an invariant set in which $D^+V = 0$ holds. Then, we have $s_i = 0$, $q_{ri} = 0$, and $\dot{q}_{ri} = \sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} = 0$ in the invariant set.

On the one hand, if $g_{ij} = 0$, it follows from (2) that $q_i = q_j$. On the other, if $g_{ij} \neq 0$, i.e., $q_i \neq q_j$, it holds that $\sum_{j\in\mathcal{N}_i} a_{ij}g_{ij} = \sum_{j\in\mathcal{N}_i} \bar{a}_{ij}(q_j - q_i)$, where $\bar{a}_{ij} = \frac{a_{ij}}{\|q_j - q_i\|}$. Thus, it follows that $(\sum_{j\in\mathcal{N}_i} \bar{a}_{ij})q_i = (\sum_{j\in\mathcal{N}_i} \bar{a}_{ij})q_j$. Then, we have $q_i = \sum_{j\in\mathcal{N}_i} \tilde{a}_{ij}q_j$, where $\tilde{a}_{ij} = \frac{\bar{a}_{ij}}{\sum_{j\in\mathcal{N}_i} \bar{a}_{ij}}$. Since $\tilde{a}_{ij} > 0$ and $\sum_{j\in\mathcal{N}_i} \tilde{a}_{ij} = 1$, q_i is in the convex hull spanned by $\{q_j\}_{j\in\mathcal{N}_i}$. However, it is impossible for all robots to locate inside a convex hull. Therefore, $\dot{V} = 0$ implies $q_i = q_j$, $\forall i \in \mathcal{V}$ and $j \in \mathcal{N}_i$.

Finally, since $s_i = 0$ and $\dot{q}_{ri} = 0$, then we have $\dot{q}_i = 0$ and $\ddot{q}_i = 0$, i.e., $M_i^{-1} - k_c s_i + \dot{q}_{ri} + (\tilde{\theta}_i + \theta_i)Y_i - (C_i\dot{q}_i + G_i)) = 0$. Therefore, $q_i(t) - q_j(t)$, $\forall i \in \mathcal{V}$, converge to zero as $t \to \infty$. Given that s_i and $\dot{q}_{ri}$ also converge to zero and Property 3 holds, the estimation errors $\tilde{\theta}_i$, $\forall i \in \mathcal{V}$, are convergent. The proof is thus completed. ■

Theorem 1 provides a solution to Problem 1 concerning systems (1), which is more general and fundamental than existing rendezvous control approaches on kinematic models, for example [1], [3], [4], [6]. In fact, robots are viewed as moving points in the kinematic models including single-integrator, double-integrator, and unicycle-type mobile robots. However, a real robot is a rigid body with specific size and shape, it is more practical to utilize dynamic model to describe robots.

IV. Case Study: Rendezvous of Wheeled Mobile Robots With Bearing Measurements

In this section, we consider a group of N dynamic wheeled mobile robots for which the formation control problems have been widely studied [27], [28]. The model of each mobile robot i is illustrated in Fig. 1 and its planar motion is described by the following kinematics

$$\dot{x}_{oi} = v_i\cos\varphi_i,\ \dot{y}_{oi} = v_i\sin\varphi_i,\ \dot{\varphi}_i = \omega_i, \tag{9}$$

for $i = 1, \ldots, N$, where v_i and ω_i are the linear and angular velocity, respectively, and $p_{oi} = [x_{oi}, y_{oi}]^{\mathrm{T}}$ and φ_i denote the position of the centroid and orientation of robot i, respectively. Define a velocity vector as $\eta_i = [v_i, \omega_i]^{\mathrm{T}}$, and the dynamics of mobile robot i can be described as

$$\bar{M}_i\dot{\eta}_i = u_i, \tag{10}$$

where $\bar{M}_i = \mathrm{diag}(m_i, I_i)$ is the inertial matrix of robot i, m_i is the mass, I_i is the moment of inertia, and u_i is the force/torque control input.

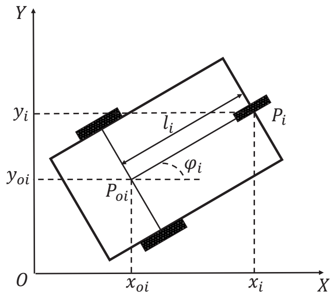

Fig. 1. Illustration of a unicycle-type wheeled mobile robot i

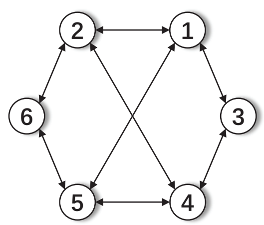

Fig. 2. Network topology

The position of robot i's head is computed as

$$p_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_{oi} \\ y_{oi} \end{bmatrix} + l_i \begin{bmatrix} \cos\varphi_i \\ \sin\varphi_i \end{bmatrix}, \tag{11}$$

where l_i is the distance between its head and centroid. Then, we obtain the velocity of robot i' head as follows,

$$\dot{p}_i = \begin{bmatrix} \cos\varphi_i & -l_i \sin\varphi_i \\ \sin\varphi_i & l_i \cos\varphi_i \end{bmatrix} \eta_i. \tag{12}$$

By inversion, we obtain

$$\eta_i = J_i(\varphi_i)\dot{p}_i, \tag{16}$$

with the Jacobian matrix

$$J_i(\varphi_i) = \begin{bmatrix} \cos\varphi_i & \sin\varphi_i \\ -l_i^{-1}\sin\varphi_i & l_i^{-1}\cos\varphi_i \end{bmatrix}. \tag{17}$$

Substituting (21) into (10) yields the dynamics of robot i' head as follows,

$$M_i(p_i)\ddot{p}_i + C_i(p_i,\dot{p}_i)\dot{p}_i = \tau_i, \tag{18}$$

where

$$\tau_i = J_i^{\mathrm{T}} u_i,\ M_i = \begin{bmatrix} M_{i11} & M_{i12} \\ M_{i21} & M_{i22} \end{bmatrix},\ C_i = \begin{bmatrix} C_{i11} & C_{i12} \\ C_{i21} & C_{i22} \end{bmatrix},$$

with

$$\begin{aligned} M_{i11} &= m_i \cos^2\varphi_i + I_i l_i^{-2}\sin^2\varphi_i, \\ M_{i22} &= m_i \sin^2\varphi_i + I_i l_i^{-2}\cos^2\varphi_i, \\ M_{i12} &= M_{i21} = (m_i - I_i l_i^{-2})\sin\varphi_i\cos\varphi_i, \end{aligned}$$

$$\begin{aligned} C_{i11} &= (I_i l_i^{-2} - m_i)\omega_i \sin\varphi_i\cos\varphi_i, \\ C_{i12} &= m_i\omega_i\cos^2\varphi_i + I_i l_i^{-2}\omega_i\sin^2\varphi_i, \\ C_{i21} &= -m_i\omega_i\sin^2\varphi_i - I_i l_i^{-2}\omega_i\cos^2\varphi_i, \\ C_{i22} &= (m_i - I_i l_i^{-2})\omega_i\sin\varphi_i\cos\varphi_i. \end{aligned} \tag{19}$$

It can be verified that Property 3 is satisfied with $\theta_i = [m_i, I_i l_i^{-2}]^{\mathrm{T}}$, and $Y_i(p_i,\dot{p}_i,a,b)$ with $a = [a_1,a_2,a_3]^{\mathrm{T}}$ and $b = [b_1,b_2,b_3]^{\mathrm{T}}$ is defined in (20).

As mentioned in Remark 2, computing $Y_i(p_i,\dot{p}_i,a,b)$ requires only the real-time orientation φ_i and angular velocity ω_i in this case, rather than the full information of position $p_i = [x_i,y_i]^{\mathrm{T}}$ and velocity $\dot{p}_i = [v_i\cos\varphi_i, v_i\sin\varphi_i]^{\mathrm{T}}$.

Note that M_i is unknown and dynamics (18) is a special case of system (1) with $G_i(q_i) = 0$, as the resistance of all robots are not considered in this case. Then, rendezvous of the networked wheeled mobile robots can be achieved, which is summarized in the following corollary.

Corollary 1: Consider N networked wheeled mobile robots in the form of (9) and (10), and a sensing network satisfying Assumption 1. Define $\dot{p}_{ri} = \sum_{j\in\mathcal{N}_i} a_{ij}\bar{g}_{ij}$, $\bar{g}_{ij} = (p_j - p_i)/\|p_j - p_i\|$, $\bar{s}_i = \dot{p}_i - \dot{p}_{ri}$, $\alpha_i = [\dot{p}_{ri}^{\mathrm{T}}, \varphi_i]^{\mathrm{T}}$, and $\beta_i = [\ddot{p}_{ri}^{\mathrm{T}}, \omega_i]^{\mathrm{T}}$. The force/torque control law

$$\begin{aligned} \tau_i &= -k_c\bar{s}_i + \dot{p}_{ri} + Y_i(p_i,\dot{p}_i,\alpha_i,\beta_i)\hat{\theta}_i, \\ \dot{\hat{\theta}}_i &= -\gamma_i Y_i^{\mathrm{T}}(p_i,\dot{p}_i,\alpha_i,\beta_i)\bar{s}_i,\ i\in\mathcal{V}, j\in\mathcal{N}_i, \end{aligned} \tag{21}$$

makes the relative displacement of robots i and j, i.e., $p_j(t) - p_i(t)$, $\forall i\in\mathcal{V},\ j\in\mathcal{N}_i$, converge to zero as $t\to\infty$ for all initial states $p_{oi}(t_0)\in\mathbb{R}^2$, $\varphi_i(t_0)\in\mathbb{R}$, and $\eta_i(t_0)\in\mathbb{R}^2$.

Note that p_i is not required in the control law (21), as (20) shows that $Y_i(p_i,\dot{p}_i,\alpha_i,\beta_i)$ is independent of p_i and $\dot{p}_i$.

The proof of Corollary 1 is omitted, since it can be done by mimicking the proof of Theorem 1.

Remark 4: The solution to rendezvous problem of multiple wheeled mobile robots does not require the synchronization of robots' orientations, and only the head positions should agree. Therefore, we define the position vector as $p_i = [x_i,y_i]^{\mathrm{T}}$ rather than $\bar{q}_i = [x_i,y_i,\varphi_i]^{\mathrm{T}}$. In fact, the normalized vector $\frac{\bar{q}_j - \bar{q}_i}{\|\bar{q}_j - \bar{q}_i\|}$ is not a bearing in practice, and is thus not considered in this case.

V. EXPERIMENTS

Laboratorial experiments on six TurtleBot3 Burger mobile robots are conducted to show the effectiveness of control law (21) for system (18). Due to the limited sensing capability of TurtleBot3 Burger mobile robot, we use VICON indoor positioning system, instead of real onboard cameras, to localize all mobile robots. Then, the instantaneous angular velocity and the relative bearings of neighboring robots, required for the control law (21), are computed and transmitted to each robot by VICON system. Although there is no

$$Y_i(p_i,\dot{p}_i,a,b) = \begin{bmatrix} (a_1+b_2b_3)\cos^2 a_3 + (a_2 - b_1b_3)\sin a_3\cos a_3 & (a_1+b_2b_3)\sin^2 a_3 + (-a_2+b_1b_3)\sin a_3\cos a_3 \\ (a_1+b_2b_3)\sin a_3\cos a_3 + (a_2-b_1b_3)\sin^2 a_3 & -(a_1+b_2b_3)\sin a_3\cos a_3 + (a_2-b_1b_3)\cos^2 a_3 \end{bmatrix}. \tag{20}$$

communication among six mobile robots, yet this VICON-based multi-robot platform is capable enough to examine the effectiveness of the proposed decentralized control law in the presence of many practical issues, such as motor offset, wear and tear of wheels, delays, and some external environmental perturbations, such as noise and friction.

In fact, it is impractical and impossible for mobile robots with sizes and shapes to share one point, that is, it holds that $p_i \neq p_j$ for all $t \geq t_0$, $i \in \mathcal{V}$, and $j \in \mathcal{N}_i$. For such real multi-robot systems, similar to the experiment conducted in [19], we set a proximity threshold $\delta > 0$ to determine whether two robots are sufficiently close. The value of δ is chosen according to the size of the mobile robots. If the distances of any two neighboring robots are less than δ, rendezvous is viewed to be achieved. Based on this, we define a practical bearing as

$$\tilde{g}_{ij} = \begin{cases} \frac{p_i - p_j}{\|p_j - p_i\|}, & \|p_j - p_i\| > \delta, \\ 0, & \|p_j - p_i\| \leq \delta. \end{cases} \tag{22}$$

The definition of $\tilde{g}_{ij}$ shows that, the relative bearings between two robots are viewed as zero if they locate sufficiently close. For each robot, if all the bearings with respect to its neighbors become zero, its force/torque controller will be set to zero. Consequently, robot stops if all its neighbors are sufficiently close. Moreover, since the underlying topology is a connected undirected graph, each robot stops if and only if all robots are sufficiently close. As a result, the networked multi-robot systems are stabilized if and only if the rendezvous is achieved.

Note that the system parameters of TurtleBot3 Burger mobile robot are unknown and they are not utilized in the controllers. The given network topology is shown in Fig. 2. The resulting linear and angular velocity driven by the virtual controller are computed by (21), which is used as the control input to the robots. Set the control gains $k_c = 5$, $\gamma_i = 0.5$, and $a_{ij} = 1$ for all $i \in \mathcal{V}, j \in \mathcal{N}_i$. Set the initial velocity and estimation values as $\dot{q}_i(t_0) = 0$ and $\hat{\theta}_i(t_0) = 0$ for all $i \in \mathcal{V}$. The initial positions and orientations of robots are set randomly. Moreover, based on the size of TurtleBot3 robots, the proximity threshold is set as $\delta = 0.6$m. To record whether the rendezvous of multi-robot systems has been achieved, we set a flag as

$$\Delta = \begin{cases} 1, & \tilde{g}_{ij} = 0, \ (i,j) \in \mathcal{E}, \\ 0, & \text{else}. \end{cases} \tag{23}$$

In Experiment 1, the initial positions and orientations of all mobile robots are set randomly. As Fig. 3 illustrates, six controlled mobile robots achieve the rendezvous in practice. Due to the size of robots, the relative distances are convergent but do not converge to zero. The trajectories during rendezvous process are shown in Fig. 4. Fig. 5 shows that total distance among neighboring agents converges to a finite value, and Fig. 6 shows that the evolution of velocities v_i. The change of rendezvous flag is given in Fig. 7.

In Experiment 2, firstly rendezvous of mobile robots is achieved and the networked multi-robot systems are stabi-

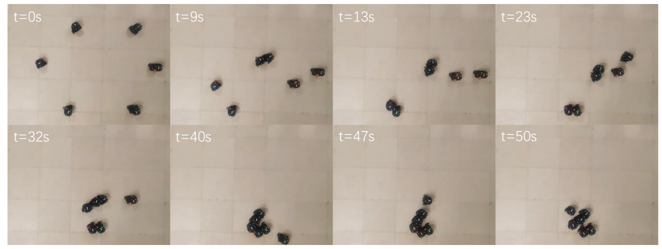

Fig. 3. Snapshots of on-site multi-robot systems in Experiment 1

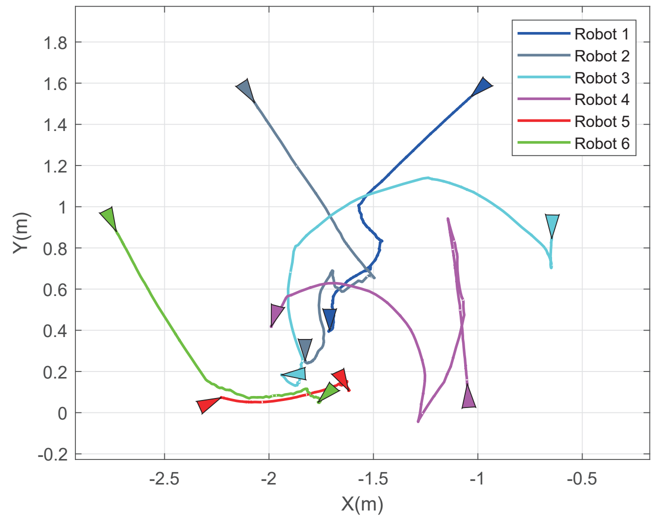

Fig. 4. Trajectories of mobile robots in Experiment 1

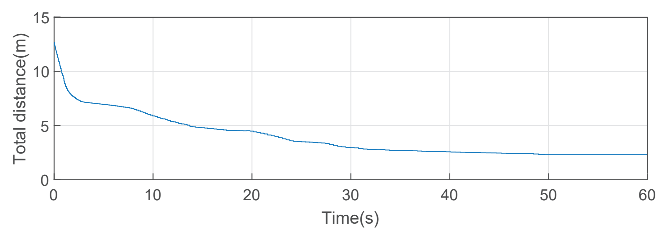

Fig. 5. Total distance $\sum_{(i,j)\in\mathcal{E}} \|p_i - p_j\|$ in Experiment 1

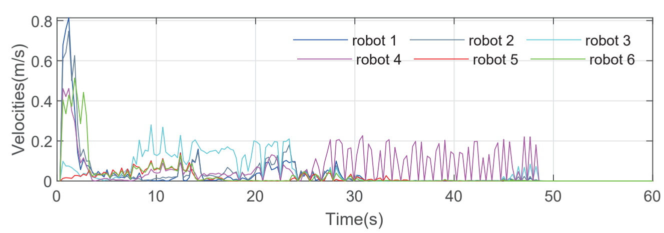

Fig. 6. Velocities v_i in Experiment 1

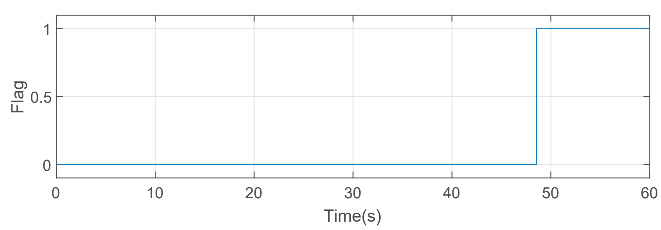

Fig. 7. Rendezvous flag Δ in Experiment 1

lized, and then we remove one robot and put it away from the original rendezvous point at $t = 28$s, as shown in Fig. 8. As Fig. 9 shows, after the interruption, the six mobile robots are converging to a new point and achieve rendezvous again, which demonstrates the global convergence of the multi-robot systems with control law (21). Fig. 10 shows that total distance among neighboring agents converges to a finite value, and Fig. 11 shows that the evolution of velocities v_i. The change of rendezvous flag is given in Fig. 12.

Remark 5: Collision avoidance is not considered in the setup of Problem 1 and the design of (4). In the existing works on rendezvous problem [3], [4], [6], the notion "Robot Merge" was proposed to theoretically handle the collision avoidance, where it was assumed that two robots would merge into a new robot if they were sufficiently close. It is obviously impossible for real robots. In [19] where the bearing-based formation control law was also implemented, a behavior-based switching algorithm was adopted to avoid collisions in the experiment. If two robots were too close, they would move away from each other, which also relies on the proximity perception. However, it is obvious that the behavior-based approach is suitable for formation problem and is essentially contradictory to rendezvous problem, since it is difficult for robots to determine whether the neighbors tend to collide or to rendezvous. To the best of our knowledge, collision avoidance in rendezvous control problem remains open. In fact, as the experimental results show, since our control law guarantees a globally convergent closed-loop system, rendezvous can still be achieved even if some collisions occur.

VI. Conclusions

In this paper, an adaptive control approach has been proposed for the networked uncertain dynamic robotic systems to solve rendezvous problem by distributed control with bearing measurements. The network topology of the multi-robot systems is described by an undirected graph, and only relative bearings are measured among the neighboring robots. The proposed approach is then used to achieve rendezvous of dynamic nonholonomic wheeled mobile robots, and is also illustrated by experiments on TurtleBot3 Burger mobile robots. In the future, formation control with bearing measurements will be investigated, conditions for connectivity preservation and collision avoidance will be studied, and a vision-based platform to achieve fully decentralized control of multi-robot systems will also be built.

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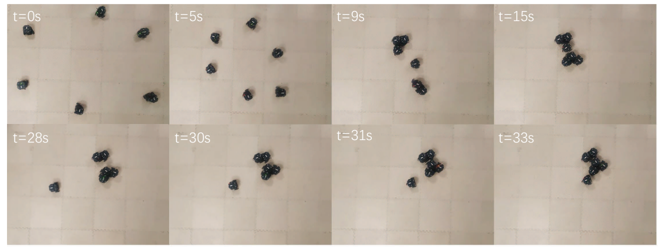

Fig. 8. Snapshots of on-site multi-robot systems in Experiment 2

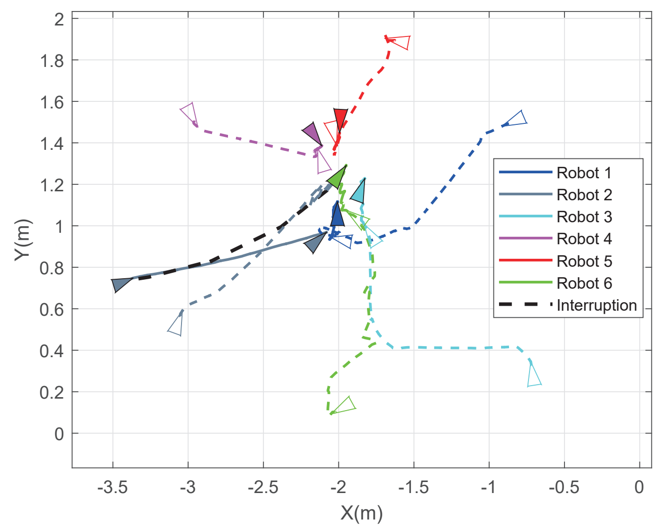

Fig. 9. Trajectories of mobile robots in Experiment 2. Dash lines: before interruption; Solid lines: after interruption

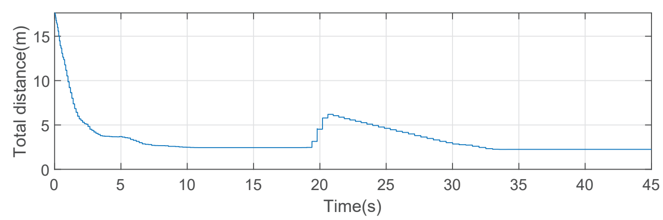

Fig. 10. Total distance $\sum_{(i,j)\in\mathcal{E}} \|p_i - p_j\|$ in Experiment 2

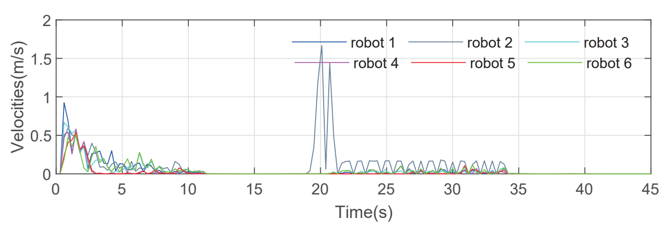

Fig. 11. Velocities v_i in Experiment 2

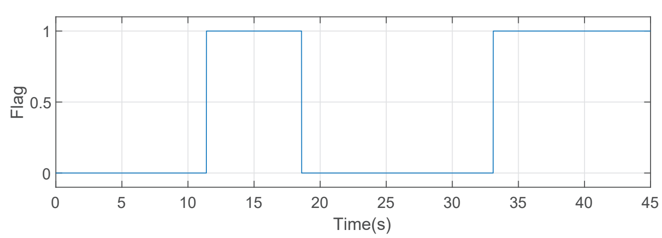

Fig. 12. Rendezvous flag Δ in Experiment 2

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