Finite-Time Cooperative Control for Bearing-Defined Leader-Following Formation of Multiple Double-Integrators

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***Abstract*—In this article, the finite-time cooperative control problem for leader-following bearing-defined formation tracking of multiagent systems with double-integrator dynamics is investigated. Different from the existing works on finite-time containment control, our objective is to make followers track leaders' trajectories and form a shape-preserving formation rather than a convex hull. The target formation is defined by both leaders' motions and bearing constraints among neighboring agents, which enables the formation not only to form and preserve a geometric pattern but also to have the ability to achieve both translational and scaling formation maneuver. To satisfy the bearing constraints, a matrix-weighted estimator/controller is developed. The finite-time stabilization of the target formation is achieved, though the matrix-weighted design makes the stability analysis complicated. Finally, an illustrative example is presented to demonstrate the effectiveness.**

***Index Terms*—Bearing-defined formation, cooperative control, finite-time control, tracking control.**

I. Introduction

FINITE-TIME control has attracted a significant amount of research attention [1]–[3], and has been applied to the consensus or formation control of multiagent systems [4]–[10]. Particularly, in [4], an estimator-based control approach was designed for leader-following consensus of double-integrators with one single leader. This approach was extended in [6] to the case with multiple leaders, which shows that all followers converge to a convex hull spanned by the leaders but the geometric pattern of the followers in the convex hull cannot be determined. However, by the containment control algorithm proposed in [6], not only the leader-following formation with a desired geometric structure cannot be formed but also it is difficult to achieve the formation maneuver.

To this end, the objective of this article is to achieve a bearing-defined target formation of double-integrators with multiple leaders in finite time. Recently, the bearing-based formation control has been a growing research interest (see for example [11]–[17]), in which the target formation is defined by the interagent bearings. For the leader-following formation, a desired geometric pattern can be uniquely determined by the leaders and the bearing constraints with respect to neighboring agents, based on the bearing rigidity theory [11] and bearing-based network localizability [18]. More importantly, since the bearing constraints are invariant to both translation and scaling of a formation, it is much easier to use the bearing-based approach to achieve translation and scaling formation maneuver than the displacement-based ones, such as [4], [6], and [19] or the distance-based ones, such as [20] and [21]. The formation maneuver could be steered by the leaders' motions, during which the geometric structure of the formation can be changed and the bearing constraints can be preserved. With such formation maneuverability, the multiagent systems can cooperatively respond to the complex environments; for instance, they can change the scale of the formation to avoid the obstacles, which has been shown in [12]. However, the finite-time control has not been achieved in all the aforementioned works on bearing-based formation control problems. In fact, the closed-loop systems with finite-time convergence usually demonstrate better disturbance rejection properties and better robustness against uncertainties. In [22], a finite-time control algorithm is developed to solve the leaderless bearing-based formation control problem where the target formation is stationary. Although bearing-based control protocols were proposed in [23] and [24] to achieve the leader-following formation control with prescribed convergent time, yet the target formations are still stationary and thus, lack of formation maneuverability. To the best of our knowledge, few works have studied the finite-time control for the bearing-defined formation tracking problem with multiple moving leaders.

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In this article, the finite-time cooperative control problem for leader-following formation tracking of multiagent systems with double-integrator dynamics is considered. The underlying network topology of the multiagent systems is described by a multiple rooted graph including an undirected subgraph, and the leaders denoted by the roots move with time-varying velocities. We develop a matrix-weighted controller based on the one in [6], such that the bearing constraints can be satisfied. Especially, a novel estimator-based control approach is developed to solve the finite-time bearing-defined formation tracking control problem.

The contributions of this article are summarized as follows. First, the proposed approach makes the multiagent systems converge to a bearing-defined formation in finite time, while only asymptotic or exponential stability was achieved in [11]–[17]. Second, in contrast to [22]–[24] where stationary target formations were achieved in finite time, the proposed approach makes followers track multiple dynamic leaders and form a moving-target formation in finite time, which enables the multiagent systems to achieve formation maneuver. Third, compared with [6] where the followers converge to a convex hull, our approach makes the multiagent systems converge to a formation with the specific geometric pattern defined by the bearing constraints. To satisfy the bearing constraints, a controller with matrix-weighted neighbor's information is developed and the finite-time stability of the closed-loop system is established. Thus, the stability analysis is much more complicated than that in [6] due to the matrix weights. Moreover, a novel estimator is designed for each follower such that its desired velocity in the bearing-defined formation can be estimated in finite time. Remarkably, different from [6] where each follower is required to compute and transmit the derivative of the estimate at the same time, the transmission of neighbors' updated values is not required for followers. Finally, unlike [6] where the positions, velocities, and accelerations of all leaders are assumed to be bounded for all time, our approach only requires the accelerations of the leaders to be bounded, which relaxes the assumption on the leaders' motion and improves the practical feasibility.

The remainder of this article is organized as follows. Section II presents the preliminaries and the problem formulation. Section III presents the proposed dynamic control law with an estimator. Section IV shows an illustrative example and finally, Section V draws the conclusion.

II. Preliminaries and Problem Formulation

A. Notations

Given a real vector $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, denote $|x| = [|x_1|, \dots, |x_n|]^T$, $\mathrm{sign}(x) = [\mathrm{sgn}(x_1), \dots, \mathrm{sgn}(x_n)]^T$, and $\mathrm{sig}^\alpha(x) = [\mathrm{sgn}(x_1)|x_1|^\alpha, \dots, \mathrm{sgn}(x_n)|x_n|^\alpha]^T$, where $\mathrm{sgn}(\cdot)$ is the standard sign function and $|\cdot|$ is the absolute value of a scalar. Let $x^\alpha = [x_1^\alpha, \dots, x_n^\alpha]^T$. Define three kinds of norms for vector x as $\|x\|_\infty = \max_{1\le j\le n}|x_j|$, $\|x\|_1 = \sum_{j=1}^n |x_j|$, and $\|x\| = \sqrt{x^Tx}$. For a matrix $A \in \mathbb{R}^{n\times n}$, let $\|A\|$ be its 2-norm. Denote $[A]_m$ and $[A]^m$, $m \in \{1, \dots, n\}$ as the mth column vector and mth row vector, respectively. Let I_d be the identity matrix, where d represents the dimension. Let $O_{m\times n}$ be the null matrix of $m \times n$ order. Throughout this article, we use the superscript "$*$" to express the corresponding desired vectors, for example, x^* denotes the desired value of x. The subscripts "l" and "f" of vectors represent the vector quantities of leaders and followers, respectively.

B. Graph Description

Consider n agents in $\mathbb{R}^d$, $d \ge 2$, consisting of n_f followers and $n_l = n - n_f \ge 2$ leaders, which are indexed by two sets $\mathcal{V}_f = \{1, \dots, n_f\}$ and $\mathcal{V}_l = \{n_f + 1, \dots, n\}$, respectively. The followers are modeled as double-integrators, that is

$$\dot{p}_i = v_i,\ \dot{v}_i = u_i,\ i \in \mathcal{V}_f \tag{1}$$

where p_i and v_i denote the position and velocity of agent i, respectively, and u_i is the acceleration and is also the control input to be designed based on the information of its neighbors. Moreover, the leaders' dynamics are given by

$$\dot{p}_j = v_j,\ j \in \mathcal{V}_l \tag{2}$$

where v_j is the time-varying velocity given *a priori* to steer the formation maneuver and it is supposed to be piecewise continuously differentiable.

The network topology of the multiagent systems is described by an n_l-rooted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ including an undirected subgraph $\mathcal{G}_f = (\mathcal{V}_f, \mathcal{E}_f)$, where $\mathcal{V} = \mathcal{V}_l \cup \mathcal{V}_f$ is the node set and $\mathcal{E} = \{(j, i) : j \ne i,\ i, j \in \mathcal{V}\}$ is the edge set, and $\mathcal{E}_f = \{(j, i) : j \ne i,\ i, j \in \mathcal{V}_f\}$ is the followers' edge set. The n_l leaders are denoted by the roots with no incoming edges because they move autonomously. As a result, graph $\mathcal{G}$ is hybrid since the edges between the leaders and the followers are unidirectional, while the edges among followers are bidirectional. The set including the neighbors of agent i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. Mapping the node i of graph $\mathcal{G}$ to p_i for all $i \in \mathcal{V}$ yields a formation $\mathcal{F}_\mathcal{G}(p)$, where $p = [p_1^T, \dots, p_n^T]^T$ is the configuration of the multiagent systems. The bearing between two neighboring agents i and j is defined as $g_{ij} = (p_j - p_i)/(\|p_j - p_i\|)$, $(i, j) \in \mathcal{E}$. Define an orthogonal projection matrix as $P_{g_{ij}^*} = I_d - g_{ij}^*(g_{ij}^*)^T$, where g_{ij}^* is the desired bearing between agents i and j. To describe the property of the graph with the desired bearing constraints, a bearing Laplacian matrix $\mathcal{B}$ [18] was defined as

$$[\mathcal{B}]_{ij} = \begin{cases} O_{d\times d}, & i \ne j, (i,j) \notin \mathcal{E} \\ -P_{g_{ij}^*}, & i \ne j, (i,j) \in \mathcal{E} \\ \sum_{k\in\mathcal{N}_i} P_{g_{ik}^*}, & i = j, i \in \mathcal{V}. \end{cases}$$

Partition $\mathcal{B}$ as $\mathcal{B} = \begin{bmatrix} \mathcal{B}_{ff} & \mathcal{B}_{fl} \\ \mathcal{B}_{lf} & \mathcal{B}_{ll} \end{bmatrix}$, where $\mathcal{B}_{lf} = O_{dn_l\times dn_f}$ and $\mathcal{B}_{ll} = O_{dn_l\times dn_l}$ hold, since the edges between the leaders and the followers are unidirectional. Moreover, since the edges among followers are bidirectional, $\mathcal{B}_{ff}$ is symmetric.

C. Problem Formulation

The target formation $\mathcal{F}_\mathcal{G}(p^*(t))$ satisfies following requirements:

1) *Bearings:* $(p_j^*(t) - p_i^*(t))/\|p_j^*(t) - p_i^*(t)\| = g_{ij}^*, (i, j) \in \mathcal{E}$,
2) *Leaders:* $p_l^*(t) = p_l(t)$, $v_l^*(t) = v_l(t)$,

where $p_l = [p_{n_f+1}^T, \dots, p_n^T]^T$ and $v_l = [v_{n_f+1}^T, \dots, v_n^T]^T$.

Then, the finite-time cooperative control problem for bearing-defined formation tracking is stated as follows.

Problem 1: Consider the multiagent systems with n agents and the network topology $\mathcal{G}$. The leaders' motions $[p_l^T, v_l^T]^T \in \mathbb{R}^{2dn_l}$ and the bearing constraints g_{ij}^*, $(i,j) \in \mathcal{E}$ are given *a priori*. For each follower i in (1) with any initial positions $p_i(t_0) \in \mathbb{R}^d$ and velocities $v_i(t_0) \in \mathbb{R}^d$, find a dynamic control law in the form of

$$\begin{aligned}\dot{\rho}_i &= \varrho(\rho_i, \rho_j)\\ u_i &= \sigma\left(p_i - p_j, v_i, \rho_i, g_{ij}^*\right), \ i \in \mathcal{V}_f, j \in \mathcal{N}_i\end{aligned} \tag{3}$$

such that the target formation $\mathcal{F}_{\mathcal{G}}(p^*(t))$ can be achieved in finite time, that is, $p_f(t) = p_f^*(t)$, $v_f(t) = v_f^*(t)$, and $g_{ij}(t) = g_{ij}^*$, $t \geq T$ for some $T \geq t_0$, where $p_f = [p_1^T, \ldots, p_{n_f}^T]^T$, $v_f = [v_1^T, \ldots, v_{n_f}^T]^T$, ρ_i is an internal state to be designed, and $\varrho(\cdot)$ and $\sigma(\cdot)$ are two sufficiently smooth functions to be designed.

To solve Problem 1, we need the following assumptions.

Assumption 1: The network $\mathcal{G}$ with bearing constraints g_{ij}^*, $(i,j) \in \mathcal{E}$, and the leaders' motion $[p_l^T, v_l^T]^T$ ensure a unique target formation $\mathcal{F}_{\mathcal{G}}(p^*(t))$.

Assumption 2: The accelerations of the leaders are bounded, that is, $\|\dot{v}_l(t)\| \leq \delta$ for $t \geq t_0$, with a positive constant δ.

Remark 1: Assumption 1 is necessary for solving a bearing-based formation control problem, since any nonunique formation cannot be guaranteed to achieve by any control approaches [18]. Only when the bearing-defined target formation is unique, then the desired position and velocity of each follower can be uniquely determined. To satisfy Assumption 1, the network $\mathcal{G}$ with bearing constraints g_{ij}^*, $(i,j) \in \mathcal{E}$ needs to be bearing rigid and every infinitesimal bearing motion involves at least one leader, based on the bearing rigidity theory [11], [18] and for details therein. Moreover, it has been shown in [11] that the graph $\mathcal{G}$ is connected if Assumption 1 is satisfied.

Remark 2: In the displacement-based formation control, the uniqueness of displacement-defined formation is independent of the network topology, since the desired position of each follower can be uniquely determined by its desired displacement with respect to the leader [19]. In the distance-based formation control, the network with distance constraints is assumed to be infinitesimally distance rigid so as to ensure a unique desired formation [20], [21]. However, both the displacement constraints and the distance constraints are not invariant to formation scaling. As a result, it is not convenient for the formation to achieve scaling maneuver via the displacement- or distance-based manners. Since the bearing constraints are invariant to both formation translation and scaling, the bearing-based formation control exhibits its merit in translational and scaling formation maneuver.

Remark 3: Assumption 2 is required for developing a finite-time estimator, as well as for ensuring the global boundedness of the closed-loop system in finite time. In [6], all of leaders' positions, velocities, and accelerations are assumed to be bounded for $t \geq t_0$, which implies that the leaders have to move in a limited region, while we only assume that the leaders' accelerations are bounded. Besides, although bearing-based control approaches were developed in [23] and [24] to achieve finite-time convergence, they can be applied to only the scenario with static leaders yet. Thus, the leaders' motion is less restrictive in this article.

III. Main Results

In this section, the finite-time cooperative control for bearing-based formation tracking problem, that is, Problem 1, is solved by an estimator-based control approach. We first design a finite-time estimator for each follower such that its desired tracking velocity is estimated. Then, we develop an acceleration controller based on the designed estimator such that the bearing-based finite-time stabilization is achieved.

A. Bearing-Based Finite-Time Estimator

To form a shape-preserving formation, each follower has its desired position and desired tracking velocity, that is, p_i^* and v_i^*, $i \in \mathcal{V}_f$. By [18, Th. 1], since the bearing-defined target formation is unique under Assumption 1, the bearing Laplacian submatrix $\mathcal{B}_{ff}$ is nonsingular, and the desired positions and velocities of the followers can be uniquely determined as

$$p_f^*(t) = -\mathcal{B}_{ff}^{-1}\mathcal{B}_{fl}p_l(t), \ \ v_f^*(t) = -\mathcal{B}_{ff}^{-1}\mathcal{B}_{fl}v_l(t). \tag{4}$$

However, both p_i^* and v_i^* are unknown to agent i. To estimate the desired tracking velocity v_i^* of each follower, we design a bearing-based finite-time estimator as

$$\begin{aligned}\dot{\hat{v}}_i = &-\nu_1 \operatorname{sign}\left(\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left(\hat{v}_i - \hat{v}_j\right)\right)\\ &- \nu_2 \operatorname{sig}^{\mu}\left(\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left(\hat{v}_i - \hat{v}_j\right)\right), \ i \in \mathcal{V}_f\end{aligned} \tag{5}$$

where the constants ν_1, ν_2, and μ satisfy $\nu_1 > \delta\|\mathcal{B}_{fl}\|/\lambda_{\min}(\mathcal{B}_{ff})$, $\nu_2 > 0$, and $\mu > 1$, respectively. By default, we set $\hat{v}_j = v_j$ for all $j \in \mathcal{V}_l$, that is, $\hat{v}_l = [\hat{v}_{n_f+1}^T, \ldots, \hat{v}_n^T]^T = v_l$ by the definition of $\mathcal{F}_{\mathcal{G}}(p^*(t))$. In other words, the leaders are required to transmit their velocities to their neighboring followers via communication, which is the same setting as that in [4], [6], and [12]. Moreover, the initial estimate of each follower $\hat{v}_i(t_0)$, $i \in \mathcal{V}_f$, can be arbitrarily set. In fact, although the value to the initial estimate $\hat{v}_i(t_0)$ can be arbitrarily chosen, in the implementation of the control law consisting of (5) and (19), we can set the initial value of $\hat{v}_i(t)$ in a feasible range, since $\hat{v}_i(t)$ is used to estimate the desired velocity v_i^*.

With the estimator (5), we have the following proposition.

Proposition 1: If Assumptions 1 and 2 are satisfied, the estimator (5) makes $\hat{v}_i$, $i \in \mathcal{V}_f$, globally converge to v_i^* in a fixed time T_0.

Proof: Define the estimation error as $e_i = \hat{v}_i - v_i^*$ and $e = [e_1^T, \ldots, e_{n_f}^T]^T$. It follows from (4) that $\mathcal{B}_{ff}v_f^* + \mathcal{B}_{fl}v_l^* = 0$ holds. Then, the desired velocities v_i^*, $i \in \mathcal{V}_f$ satisfy

$$\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(v_i^* - v_j^*) = 0. \tag{6}$$

Then, we have

$$\begin{aligned}\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\hat{v}_i-\hat{v}_j) &= \sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\Big(\big(e_i+v_i^*\big)-\big(e_j+v_j^*\big)\Big)\\ &= \sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\Big(\big(e_i-e_j\big)+\big(v_i^*-v_j^*\big)\Big)\\ &= \mathcal{B}_{ff}e+\mathcal{B}_{fl}\big(\hat{v}_l-v_l^*\big).\end{aligned}\tag{7}$$

Since we set $\hat{v}_l=v_l^*=v_l$, then, it holds that

$$\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\hat{v}_i-\hat{v}_j)=\mathcal{B}_{ff}e.\tag{8}$$

Thus, the estimation error system is written as

$$\dot{e}=-\nu_1\operatorname{sign}(\mathcal{B}_{ff}e)-\nu_2\operatorname{sig}^{\mu}(\mathcal{B}_{ff}e)-\dot{v}_f^*.\tag{9}$$

Consider a Lyapunov function candidate

$$U=\nu_1\mathbf{1}_{dn_f}^T|\mathcal{B}_{ff}e|+\frac{\nu_2}{\mu+1}\mathbf{1}_{dn_f}^T|\mathcal{B}_{ff}e|^{\mu+1}\tag{10}$$

which is differentiable almost everywhere expect in the set $S=\{e:\prod_{i=1}^{n_f}e_i=0\}$. Taking the time derivative of U along the trajectory of system (9) gives

$$\begin{aligned}\dot{U}=&-\Big[\nu_1\operatorname{sign}(\mathcal{B}_{ff}e)+\nu_2\operatorname{sig}^{\mu}(\mathcal{B}_{ff}e)+\dot{v}_f^*\Big]^T\\ &\times\mathcal{B}_{ff}\big[\nu_1\operatorname{sign}(\mathcal{B}_{ff})+\nu_2\operatorname{sig}^{\mu}(\mathcal{B}_{ff}e)\big].\end{aligned}\tag{11}$$

Let $y=\mathcal{B}_{ff}e$. Under Assumption 1, the graph $\mathcal{G}$ is connected [11]. For the connected graph $\mathcal{G}$ with an undirected subgraph $\mathcal{G}_f$, there exists at least one leader that has a path to each follower, which makes the matrix $\mathcal{B}_{ff}$ positive definite by [6, Lemma 5]. Moreover, it follows from (4) that $\mathcal{B}_{ff}\dot{v}_f^*=-\mathcal{B}_{fl}\dot{v}_l$

$$\begin{aligned}\dot{U}\le&-\lambda_{\min}(\mathcal{B}_{ff})\big\|\nu_1\operatorname{sign}(y)+\nu_2\operatorname{sig}^{\mu}(y)\big\|^2\\ &-\big[\nu_1\operatorname{sign}(y)+\nu_2\operatorname{sig}^{\mu}(y)\big]^T\mathcal{B}_{fl}\dot{v}_l\\ \le&-\lambda_{\min}(\mathcal{B}_{ff})\sum_{k=1}^{dn_f}\Big(\nu_1^2+2\nu_1\nu_2|y_k|^{\mu}+\nu_2^2|y_k|^{2\mu}\Big)\\ &+\delta\|\mathcal{B}_{fl}\|\sum_{k=1}^{dn_f}\big(\nu_1+\nu_2|y_k|^{\mu}\big)\le-\epsilon_1\tilde{U}\end{aligned}\tag{12}$$

where $\epsilon_1=\min\{\nu_2\lambda_{\min}(\mathcal{B}_{ff}),\ \nu_1(\nu_1\lambda_{\min}(\mathcal{B}_{ff})-\delta\|\mathcal{B}_{fl}\|),\ \nu_2(2\nu_1\lambda_{\min}(\mathcal{B}_{ff})-\delta\|\mathcal{B}_{fl}\|)\}$, and

$$\tilde{U}=\sum_{k=1}^{dn_f}\Big(|y_k|^{2\mu}+|y_k|^{\mu}+1\Big).\tag{13}$$

Choose $\nu_1>\delta\|\mathcal{B}_{fl}\|/\lambda_{\min}(\mathcal{B}_{ff})$. Then, we have $\epsilon_1>0$. By [25, Lemma 5], it holds that

$$\tilde{U}\ge\frac{1}{2}(2n_f)^{\frac{1-\mu}{1+\mu}}\bar{U}^{\frac{2\mu}{\mu+1}}+\frac{1}{2}\bar{U}^{\frac{\mu}{\mu+1}}\tag{14}$$

where $\bar{U}=\sum_{k=1}^{dn_f}(|y_k|^{\mu+1}+|y_k|)$. It follows from (10) that

$$U=\sum_{k=1}^{dn_f}\left(\nu_1|y_k|+\frac{\nu_2}{\mu+1}|y_k|^{\mu+1}\right)\le\epsilon_2\bar{U}\tag{15}$$

where $\epsilon_2=\max\{\nu_1,\nu_2/(\mu+1)\}$. By (12), (14), and (15), we have

$$\dot{U}\le-\varsigma_1U^{\frac{2\mu}{\mu+1}}-\varsigma_2U^{\frac{\mu}{\mu+1}}\tag{16}$$

where $\varsigma_1=(\epsilon_1(2dn_f)^{(1-\mu)/(1+\mu)})/(2\epsilon_2^{(2\mu)/(1+\mu)})$ and $\varsigma_2=\epsilon_1/2\epsilon_2^{\mu/(\mu+1)}$. By [26, Th. 5], system (9) is finite-time stable with the settling time $T_0=(\mu+1)/\varsigma_1(\mu-1)+(\mu+1)/\varsigma_2$. Therefore, $e(t)$ converges to zero in the fixed time T_0, that is, $\hat{v}_f$ converges to v_f^* in the fixed time T_0, where $\hat{v}_f=[\hat{v}_1^T,\ldots,\hat{v}_{n_f}^T]^T$. ∎

Remark 4: The design of estimator (5) is inspired by [27]. One requirement of implementing this estimator is that the parameter ν_1 needs to be sufficiently large such that $\nu_1>\delta\|\mathcal{B}_{fl}\|/\lambda_{\min}(\mathcal{B}_{ff})$ is satisfied. In fact, the estimator (5) can be incorporated with the adaptive protocol [28] to achieve a fully distributed estimation. Moreover, the design of estimator (5) only requires each agent to transmit the estimated desired velocities $\hat{v}_j$, $j\in\mathcal{V}_f$, to its neighbors. In [6], the transmission of $\dot{\hat{v}}_j$, $j\in\mathcal{V}_f$, is, in addition, required. Given the fact that it is impractical to conduct the computation and transmission of the information $\dot{\hat{v}}_j$ at the same time, the estimator (5) exhibits its merit over that in [6].

Remark 5: There may exist chattering phenomenon near the equilibrium, since the sign function $\operatorname{sign}(\cdot)$ on the right-hand side of the sliding-mode-like estimator (5) is non-Lipschitz, even though it would not influence the finite-time convergence of the estimate error system (9). Note that the continuous function $\operatorname{sig}(\cdot)$ in (5) is chattering free according to [29]. To weaken the chattering brought by the sign function $\operatorname{sign}(\cdot)$, the estimator (5) can be modified as

$$\begin{aligned}\dot{\hat{v}}_i=&-\nu_1\operatorname{sat}_{\epsilon}\left(\operatorname{sign}\left(\sum_{j\in\mathcal{N}_i}P_{g_{ij}^*}(\hat{v}_i-\hat{v}_j)\right)\right)\\ &-\nu_2\operatorname{sig}^{\mu}\left(\sum_{j\in\mathcal{N}_i}P_{g_{ij}^*}(\hat{v}_i-\hat{v}_j)\right)\end{aligned}\tag{17}$$

where $\epsilon\ge1$ is a given positive constant, and $\operatorname{sat}_{\epsilon}(\cdot)$ is a saturation function defined by

$$\operatorname{sat}_{\epsilon}(x)=\begin{cases}x, & \|x\|\le\epsilon\\ \epsilon\operatorname{sign}(x), & \|x\|>\epsilon.\end{cases}\tag{18}$$

By mimicking the proof of [5, Th. 3], it can be shown that the resulting estimate error system under (17) is also finite-time convergent. Due to the page limit, the detail is omitted.

B. Finite-Time Tracking Subject to Bearing Constraints

Based on the estimator (5), we further develop a bearing-based acceleration controller as follows:

$$u_i=\dot{\hat{v}}_i-k_2\left(\big(v_i-\hat{v}_i\big)^{1/\alpha}+k_1^{1/\alpha}\sum_{j\in\mathcal{N}_i}P_{g_{ij}^*}(p_i-p_j)\right)^{2\alpha-1}\tag{19}$$

where k_1 and k_2 are two positive control gains satisfying $k_1\ge2^{1-\alpha}/(\alpha+1)+d\alpha(n+n_f)/(\alpha+1)+k_3$, $k_2\ge(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}(2^{1-\alpha}/(\alpha+1)+(d(2^{1-\alpha}\alpha(n+1)+(n+n_f)(k_1+2^{1-\alpha}))/k_1(\alpha+1))+k_3)$,

$k_3 > 0$, and the constant α satisfies $1/2 < \alpha = \alpha_1/\alpha_2 < 1$, with α_1 and α_2 being positive odd integers.

Before the finite-time stabilization of the closed-loop system is presented, we first provide the following proposition, which shows that the trajectories of the closed-loop system are bounded during the time period $[t_0, t_0 + T_0]$.

Proposition 2: Under Assumptions 1 and 2, for the trajectories of the multiagent systems (1) driven by the control law consisting of (5) and (19), $p_f(t)$ and $v_f(t)$ are bounded for $t \in [t_0, t_0 + T_0]$.

Proof: The proof is given in Appendix B. ■

Next, define the tracking errors as $\bar{p}_i = p_i - p_i^*$, $\bar{v}_i = v_i - v_i^*$, and the compact tracking errors as $\bar{p} = [\bar{p}_1^T, \ldots, \bar{p}_{n_f}^T]^T$, $\bar{v} = [\bar{v}_1^T, \ldots, \bar{v}_{n_f}^T]^T$. By Propositions 1 and 2, we have $\hat{v}_i = v_i^*$ for $t \geq t_0 + T_0$. Similar to (7), we have

$$\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\bar{p}_i - \bar{p}_j) = \sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(p_i - p_j).$$

With the facts above, substituting (19) into (1) yields the following error system:

$$\begin{aligned}\dot{\bar{p}}_i &= \bar{v}_i\\ \dot{\bar{v}}_i &= -k_2\left(\bar{v}_i^{1/\alpha} + k_1^{1/\alpha}\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\bar{p}_i - \bar{p}_j)\right)^{2\alpha-1}.\end{aligned} \tag{20}$$

Now, we present the main result of this article, which establishes the finite-time stability of the resulting closed-loop system.

Theorem 1: If Assumptions 1 and 2 are satisfied, the control law consisting of (5) and (19) makes the tracking errors $\bar{p}_f$ and $\bar{v}_f$ of system (20) converge to zero in finite time.

Proof: Consider the Lyapunov function $V = V_0 + \sum_{i=1}^{n_f} V_i$, with

$$V_0 = \frac{1}{2}\bar{p}_f^T \mathcal{B}_{ff}\bar{p}_f,\quad V_i = \sum_{m=1}^{d} V_{im} \tag{21}$$

$$V_{im} = \frac{1}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}\int_{\tilde{v}_{im}}^{\bar{v}_{im}} (s^{1/\alpha} - \tilde{v}_{im}^{1/\alpha})^{2-\alpha}\mathrm{d}s \tag{22}$$

$$\tilde{v}_i = -k_1 w_i^\alpha,\quad w_i = \sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\bar{p}_i - \bar{p}_j). \tag{23}$$

First, taking the time derivative of V_0 along the trajectories of system (20) gives

$$\begin{aligned}\dot{V}_0 &= \bar{p}_f^T\mathcal{B}_{ff}\dot{\bar{p}}_f = \left(\mathcal{B}_{ff}\bar{p}_f\right)^T\bar{v}_f\\ &= \sum_{i=1}^{n_f}\left[\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\bar{p}_i - \bar{p}_j)\right]^T\bar{v}_i = \sum_{i=1}^{n_f} w_i^T\bar{v}_i\end{aligned} \tag{24}$$

since $\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(\bar{p}_i - \bar{p}_j) = \mathcal{B}_{ff}\bar{p}_f + \mathcal{B}_{fl}\bar{p}_l$ and $\mathcal{B}_{fl}\bar{p}_l = \mathcal{B}_{fl}(p_l - p_l^*) = 0$. Then, we have

$$\begin{aligned}\dot{V}_0 &= \sum_{i=1}^{n_f} w_i^T(\tilde{v}_i + (\bar{v}_i - \tilde{v}_i))\\ &= -k_1\sum_{i=1}^{n_f} w_i^T w_i^\alpha + \sum_{i=1}^{n_f} w_i^T(\bar{v}_i - \tilde{v}_i)\\ &\leq -k_1\sum_{i=1}^{n_f}\sum_{m=1}^{d} w_{im}^{\alpha+1} + \sum_{i=1}^{n_f}\sum_{m=1}^{d}|w_{im}||\bar{v}_{im} - \tilde{v}_{im}|.\end{aligned} \tag{25}$$

Let $\xi_i = \bar{v}_i^{1/\alpha} - \tilde{v}_i^{1/\alpha} = [\xi_{i1}, \ldots, \xi_{id}]^T$. It follows from Lemma 1 that

$$|\bar{v}_{im} - \tilde{v}_{im}| \leq 2^{1-\alpha}\left|\bar{v}_{im}^{1/\alpha} - \tilde{v}_{im}^{1/\alpha}\right|^\alpha = 2^{1-\alpha}|\xi_{im}|^\alpha. \tag{26}$$

Then, we have

$$\begin{aligned}\dot{V}_0 &\leq -k_1\sum_{i=1}^{n_f}\sum_{m=1}^{d} w_{im}^{\alpha+1} + 2^{1-\alpha}\sum_{i=1}^{n_f}\sum_{m=1}^{d}\left(\frac{w_{im}^{\alpha+1}}{\alpha+1} + \frac{\alpha\xi_{im}^{\alpha+1}}{\alpha+1}\right)\\ &= \sum_{i=1}^{n_f}\sum_{m=1}^{d}\left(-\left(k_1 - \frac{2^{1-\alpha}}{\alpha+1}\right)w_{im}^{\alpha+1} + \frac{\alpha 2^{1-\alpha}}{\alpha+1}\xi_{im}^{\alpha+1}\right)\end{aligned} \tag{27}$$

where the inequality is obtained by Lemma 2.

Second, taking the time derivative of V_{im} along the trajectories of system (20) gives

$$\begin{aligned}\dot{V}_{im} = &-\frac{1}{2^{1-\alpha}k_1^{1+1/\alpha}}\frac{\mathrm{d}\tilde{v}_{im}^{1/\alpha}}{\mathrm{d}t}\int_{\tilde{v}_{im}}^{\bar{v}_{im}}\left(s^{1/\alpha} - \tilde{v}_{im}^{1/\alpha}\right)^{1-\alpha}\mathrm{d}s\\ &+ \frac{\xi_{im}^{2-\alpha}\bar{u}_{im}}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}\end{aligned} \tag{28}$$

where $\bar{u}_{im}$ is the mth component of $\bar{u}_i$, and $\bar{u}_i = \dot{\bar{v}}_i$. Moreover, we have the following facts that:

$$\int_{\tilde{v}_{im}}^{\bar{v}_{im}}\left(s^{1/\alpha} - \tilde{v}_{im}^{1/\alpha}\right)^{1-\alpha}\mathrm{d}s \leq |\bar{v}_{im} - \tilde{v}_{im}||\xi_{im}|^{1-\alpha} \leq 2^{1-\alpha}|\xi_{im}|$$

and

$$\begin{aligned}\frac{\mathrm{d}\tilde{v}_{im}^{1/\alpha}}{\mathrm{d}t} &= -k_1^{1/\alpha}\sum_{j\in\mathcal{N}_i}\left[P_{g_{ij}^*}(\bar{v}_i - \bar{v}_j)\right]_m\\ &= -k_1^{1/\alpha}\left(\sum_{j\in\mathcal{N}_i}\left[P_{g_{ij}^*}\right]^m\bar{v}_i - \sum_{j\in\mathcal{N}_i}\left[P_{g_{ij}^*}\right]^m\bar{v}_j\right)\\ &\leq k_1^{1/\alpha}\sum_{j\in\mathcal{N}_i}\left\|\left[P_{g_{ij}^*}\right]^m\right\|_\infty\|\bar{v}_i\|_1\\ &\quad + k_1^{1/\alpha}\sum_{j=1}^{n_f}\left\|\left[P_{g_{ij}^*}\right]^m\right\|_\infty\left\|\bar{v}_j\right\|_1.\end{aligned}$$

Since $P_{g_{ij}^*}$ is an orthogonal projection matrix for all $(i,j) \in \mathcal{E}$, it holds that $\|[P_{g_{ij}^*}]^m\|_\infty \leq \|[P_{g_{ij}^*}]^m\| \leq \|P_{g_{ij}^*}\| = 1$. It follows that

$$\frac{\mathrm{d}\tilde{v}_{im}^{1/\alpha}}{\mathrm{d}t} \leq k_1^{1/\alpha}\left(n\|\bar{v}_i\|_1 + \sum_{j=1}^{n_f}\left\|\bar{v}_j\right\|_1\right). \tag{29}$$

Then, we have

$$\begin{aligned}\dot{V}_{im} \leq &\frac{1}{k_1}\left(n\|\bar{v}_i\|_1 + \sum_{j=1}^{n_f}\left\|\bar{v}_j\right\|_1\right)|\xi_{im}|\\ &+ \frac{\xi_{im}^{2-\alpha}\bar{u}_{im}}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}.\end{aligned} \tag{30}$$

Consider $\|\bar{v}_i\|_1 = \sum_{\iota=1}^{d}|\bar{v}_{i\iota}|$, where $v_{i\iota}$ is the ι-component of $\bar{v}_i$. By Lemma 1, $|\bar{v}_{i\iota}|$ satisfies

$$|\bar{v}_{i\iota}| \leq |\tilde{v}_{i\iota}| + |\bar{v}_{i\iota} - \tilde{v}_{i\iota}| \leq k_1|w_{i\iota}|^\alpha + 2^{1-\alpha}|\xi_{i\iota}|^\alpha.$$

Then, it follows from Lemma 2 that:

$$\begin{aligned}\|\bar{v}_i\|_1|\xi_{im}| &\leq \sum_{\iota=1}^{d}\Big(k_1|w_{i\iota}|^{\alpha} + 2^{1-\alpha}|\xi_{i\iota}|^{\alpha}\Big)|\xi_{im}| \\ &\leq \sum_{\iota=1}^{d}\Big(\frac{k_1\alpha}{\alpha+1}w_{i\iota}^{\alpha+1} + \frac{2^{1-\alpha}\alpha}{\alpha+1}\xi_{i\iota}^{\alpha+1} \\ &\qquad + \frac{k_1+2^{1-\alpha}}{\alpha+1}\xi_{im}^{\alpha+1}\Big). \end{aligned} \tag{31}$$

As $\bar{u}_i = -k_2\xi_i^{2\alpha-1}$, we have $\bar{u}_{im} = -k_2\xi_{im}^{2\alpha-1}$. By (27), (31), and (32), we obtain

$$\begin{aligned}\dot{V}_{im} \leq &\sum_{\iota=1}^{d}\Big(\frac{\alpha n}{\alpha+1}w_{i\iota}^{\alpha+1} + \frac{2^{1-\alpha}\alpha n}{k_1(\alpha+1)}\xi_{i\iota}^{\alpha+1}\Big) \\ &+ \frac{\alpha}{\alpha+1}\sum_{\iota=1}^{d}\sum_{j=1}^{n_f}w_{j\iota}^{\alpha+1} + \frac{2^{1-\alpha}\alpha}{k_1(\alpha+1)}\sum_{\iota=1}^{d}\sum_{j=1}^{n_f}\xi_{j\iota}^{\alpha+1} \\ &+ \left(\frac{d(n+n_f)(k_1+2^{1-\alpha})}{k_1(1+\alpha)} + \frac{-k_2}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}\right)\xi_{im}^{\alpha+1}.\end{aligned}$$

Then, it follows from $\dot{V}_i = \sum_{m=1}^{d}\dot{V}_{im}$ that:

$$\begin{aligned}\dot{V}_i \leq\ &d\sum_{\iota=1}^{d}\Big(\frac{\alpha n}{\alpha+1}w_{i\iota}^{\alpha+1} + \frac{2^{1-\alpha}\alpha n}{k_1(\alpha+1)}\xi_{i\iota}^{\alpha+1}\Big) \\ &+ \frac{d\alpha}{\alpha+1}\sum_{\iota=1}^{d}\sum_{j=1}^{n_f}w_{j\iota}^{\alpha+1} + \frac{2^{1-\alpha}d\alpha}{k_1(\alpha+1)}\sum_{\iota=1}^{d}\sum_{j=1}^{n_f}\xi_{j\iota}^{\alpha+1} \\ &+ \sum_{m=1}^{d}\left(\frac{d(n+n_f)(k_1+2^{1-\alpha})}{k_1(1+\alpha)}\right)\xi_{im}^{\alpha+1} \\ &+ \sum_{m=1}^{d}\left(\frac{-k_2}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}\right)\xi_{im}^{\alpha+1}.\end{aligned}$$

As a result, with $\dot{V} = \dot{V}_0 + \sum_{i=1}^{n_f}\dot{V}_{im}$, we have

$$\dot{V} \leq C_1\sum_{i=1}^{n_f}\sum_{m=1}^{d}w_{im}^{\alpha+1} + C_2\sum_{i=1}^{n_f}\sum_{m=1}^{d}\xi_{im}^{\alpha+1} \tag{32}$$

where

$$\begin{aligned}C_1 &= -k_1 + \frac{2^{1-\alpha}}{\alpha+1} + \frac{d\alpha(n+n_f)}{\alpha+1} \\ C_2 &= \frac{d\big(2^{1-\alpha}\alpha(n+1) + (n+n_f)(k_1+2^{1-\alpha})\big)}{k_1(\alpha+1)} + \frac{2^{1-\alpha}\alpha}{\alpha+1} \\ &\quad - \frac{k_2}{(2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}}.\end{aligned}$$

Because k_1, k_2, and k_3 satisfy

$$\begin{aligned}k_1 &\geq \frac{2^{1-\alpha}}{\alpha+1} + \frac{d\alpha(n+n_f)\alpha+1}{+}k_3 \\ k_2 &\geq (2-\alpha)2^{1-\alpha}k_1^{1+1/\alpha}(C_3+k_3) \\ C_3 &= \frac{2^{1-\alpha}}{\alpha+1} + \frac{d\big(2^{1-\alpha}\alpha(n+1) + (n+n_f)(k_1+2^{1-\alpha})\big)}{k_1(\alpha+1)} \\ k_3 &> 0\end{aligned}$$

it holds that

$$\dot{V} \leq -k_3\sum_{i=1}^{n}\sum_{m=1}^{d}\Big(w_{im}^{\alpha+1} + \xi_{im}^{\alpha+1}\Big) \tag{33}$$

which is negative definite since $\alpha+1$ satisfies $\alpha_1+1 = (\alpha_1+\alpha_2)/\alpha_2$ and $\alpha_1+\alpha_2$ is even.

Third, it follows from (23) that: $w_f = [w_1^T, \ldots, w_{n_f}^T]^T = \mathcal{B}_{ff}\bar{p}_f$. Accordingly, it holds that

$$\|w_f\|^2 = \sum_{i=1}^{n_f}w_i^Tw_i = \bar{p}_f^T\mathcal{B}_{ff}^2\bar{p}_f.$$

Then, we have

$$\|w_f\|^2 \geq \lambda_{\min}\Big(\mathcal{B}_{ff}^2\Big)\bar{p}_f^T\bar{p}_f \geq \frac{2\lambda_{\min}\Big(\mathcal{B}_{ff}^2\Big)V_0}{\lambda_{\max}(\mathcal{B}_{ff})}. \tag{34}$$

Moreover, it follows from (22) and (26) that:

$$V_i \leq \frac{1}{(2-\alpha)k_1^{1+1/\alpha}}\sum_{m=1}^{d}|\xi_{im}|^2. \tag{35}$$

By (34) and (35), we obtain

$$\begin{aligned}V &\leq c\sum_{i=1}^{n_f}\sum_{m=1}^{d}\Big(w_{im}^2 + \xi_{im}^2\Big) \\ &\leq c\sum_{i=1}^{n_f}\sum_{l=1}^{d}\Big(w_{il}^{\alpha+1} + \xi_{il}^{\alpha+1}\Big)^{2/(\alpha+1)}\end{aligned} \tag{36}$$

where $c = \max\{\lambda_{\max}(\mathcal{B}_{ff})/2\lambda_{\min}(\mathcal{B}_{ff}^2), 1/(2-\alpha)k_1^{1+1/\alpha}\}$, and the last inequality is due to Lemma 1.

Finally, by (32) and (36), we have

$$\dot{V} + \frac{k_3}{c^{(\alpha+1)/2}}V^{(\alpha+1)/2} \leq 0 \tag{37}$$

where $(\alpha+1)/2 \in (0,1)$ due to $\alpha < 1$. By [1, Th. 4.2], the tracking errors $\bar{p}_f$ and $\bar{v}_f$ of system (20) converge to zero in the finite time $T \leq V(t_0)^{(1-\alpha)/2}/(c(1-\alpha)/2)$. The proof is thus completed. ■

Remark 6: The design of the controller (19) is motivated by [6] and [12]. To implement the control law consisting of (5) and (19), all the agents are supposed to know a common reference direction. Nevertheless, a common origin is not required and a universal coordinate system is thus not necessary. Then, the relative positions $p_i - p_j$ can be directly measured by their onboard campass/IMU, camera, and lidar. The controller (19) only requires the local measurements and the communication among neighboring agents and thus, can be implemented in a distributed manner. Compared with the containment control problem in [6], where followers converge to a dynamic convex hull formulated by leaders, we require the followers to form a bearing-defined shape-preserving formation with respect to the leaders. Accordingly, we design a dynamic control law consisting of matrix-weighted estimator (5) and controller (19) to achieve the bearing-defined target formation. It is the matrix-weighted terms that increase the difficulties in the establishment of the finite-time-stability. More complicated operations of 1-norm and ∞-norm of vectors and matrices have to be handled.

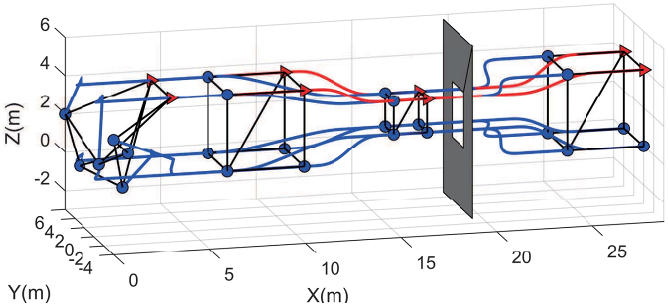

Fig. 1. Trajectories of leaders and followers.

Remark 7: The error system (20) is obtained based on the estimation result $\hat{v}_i = v_i^*$ for $t \geq T_0$ by Proposition 1. Proposition 2 shows that the states $p_f(t)$ and $v_f(t)$ do not diverge for all $t \in [t_0, t_0 + T_0]$, which guarantees that the initial states of error system (20), that is, $\bar{p}_i(t_0 + T_0)$ and $\bar{v}_i(t_0+T_0)$, $i \in \mathcal{V}_f$, are bounded. In [6, Proposition 2], the positions and velocities of followers are bounded for $t \geq t_0$ under the assumption that all of leaders' positions, velocities, and accelerations are bounded for $t \geq t_0$. However, this assumption is limited as Remark 1 states. In fact, we have shown that Assumption 2 is sufficient to establish Theorem 1. Finally, it is noted that the final settling time T_f of the closed-loop system driven by the control laws consisting of (5) and (19) satisfies $T_f \leq t_0 + T_0 + T$, which can be regulated by tuning the constants k_1, k_2, and α.

Remark 8: For the network topology of the multiagent systems, the subgraph among all followers $\mathcal{G}_f$ is assumed to be undirected since it is necessary to guarantee the positive definiteness of $\mathcal{B}_{ff}$. The positive-definite matrix $\mathcal{B}_{ff}$ is used to construct Lyapunov functions (10) and (21) in the proofs of Proposition 1 and Theorem 1. While if G_f became a directed graph or even a strongly connected graph, $\mathcal{B}_{ff}$ would not necessarily be symmetric, letting alone ensure it is positive definite. The bearing-based formation control problem of multiagent systems under a directed network will be further studied in our future work.

IV. Illustrative Example

In this section, we present a simulation example to illustrate the effectiveness of the control law consisting of (5) and (19) for the followers (1). The multiagent systems consist of the followers 1–6, the leaders 7 and 8, and the 2-rooted graph G. The target formation is a cube, defined by the bearing constraints $g^*_{3,2} = g^*_{4,1} = -g^*_{7,6} = g^*_{8,5} = [0, 1, 0]^T$, $g^*_{4,3} = g^*_{5,6} = -g^*_{7,8} = [1, 0, 0]^T$, $g^*_{3,7} = g^*_{4,8} = -g^*_{5,1} = -g^*_{6,2} = [0, 0, -1]^T$, and $g^*_{8,2} = (1/\sqrt{3})[1, 1, 1]^T$. The leaders' initial positions are $p_1(t_0) = [4, 0, 4]^T$ and $p_2(t_0) = [4, 4, 4]^T$. All the agents are stationary at the initial time. To avoid the obstacle, the leaders are associated with the following accelerations so as to steer the multiagent formation maneuver:

$$0 < t \leq 10 : \dot{v}_1 = \left[0.03 \sin(t/4), 0, 0\right]^T$$
$$\dot{v}_2 = \left[0.03 \sin(t/4), 0, 0\right]^T$$

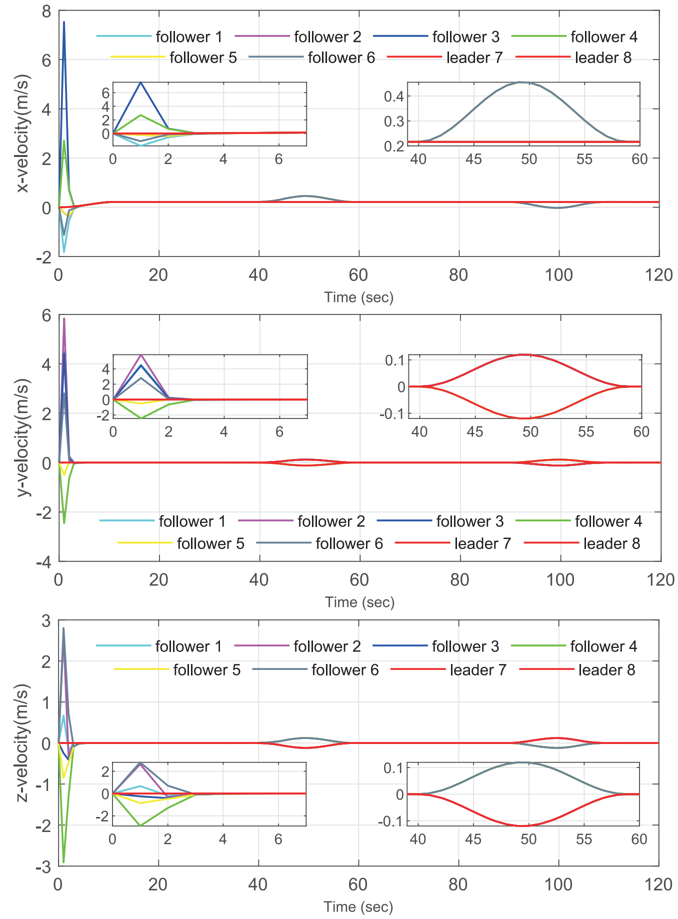

Fig. 2. Velocity of each follower v_i.

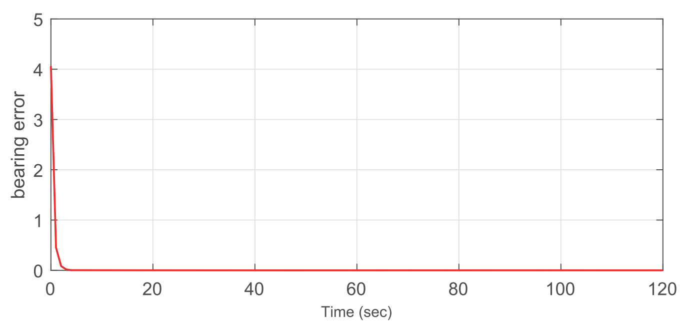

Fig. 3. Bearing error $\sum_{(i,j)\in\mathcal{E}} \|g_{ij} - g^*_{ij}\|$.

$$10 < t \leq 40 : \dot{v}_1 = [0, 0, 0]^T, \ \dot{v}_2 = [0, 0, 0]^T$$
$$40 < t \leq 60 : \dot{v}_1 = [0, 0.02 \sin((t-40)/3), -0.02 \sin((t-40)/3)]^T$$
$$\dot{v}_2 = [0, -0.02 \sin((t-40)/3), -0.02 \sin((t-40)/3)]^T$$
$$60 < t \leq 90 : \dot{v}_1 = [0, 0, 0]^T, \ \dot{v}_2 = [0, 0, 0]^T$$
$$90 < t \leq 110 :$$
$$\dot{v}_1 = [0, -0.02 \sin((t-40)/3), 0.02 \sin((t-40)/3)]^T$$
$$\dot{v}_2 = [0, 0.02 \sin((t-40)/3), 0.02 \sin((t-40)/3)]^T$$
$$t > 110 : \dot{v}_1 = [0, 0, 0]^T, \ \dot{v}_2 = [0, 0, 0]^T.$$

The initial positions of the followers are set randomly. Set $\mu = 1.2$, $\nu_1 = 10$, $\nu_2 = 2$, $k_1 = 10$, $k_2 = 5$, and $\alpha = 9/11$.

Fig. 1 illustrates that the multiagent systems converge to the desired cube formation, which is able to achieve the translational and scaling formation maneuver. At $t = 40$s and 90s, the formation achieved the scaling maneuver and avoid the obstacle successfully.

The change of the velocity of each agent is presented in Fig. 2. As is shown in Fig. 3, the bearing error converges to zero in finite time $t = 3$s, and remains zero during the translational and scaling formation maneuver.

V. Conclusion

In this article, a distributed estimator-based control approach has been proposed for the multiagent systems consisting of multiple leaders, followers with double-integrator dynamics, and the underlying multiple rooted graph including an undirected subgraph, to achieve a bearing-defined target formation in finite time. Different from the convex hull, the bearing-defined formation not only has the desired geometric structure but also is capable of formation maneuverability. Simulation results are also presented to illustrate the effectiveness of our control approach.

There are some directions for future work. First, we hope to investigate the bearing-based formation control problem over more complicated network topology, such as a switching graph or a directed graph. Second, some practical issues, including the conditions for collision avoidance and connectivity preservation, will be considered. Last but not least, it is of much practicability to employ the event-triggered mechanism to achieve bearing-based formation control against cyberattacks [30], [31] in the future.

Appendix A
Lemmas

Lemma 1 [32, Lemma 2.3]: For any real numbers, x_i, $i = 1, \ldots, n$ and $0 < b \leq 1$, the following inequality holds, $(\sum_{i=1}^{n} |x_i|)^b \leq \sum_{i=1}^{n} |x_i|^b$. If $b = b_1/b_2 \leq 1$, where $b_1 > 0$ and $b_2 > 0$ are odd integers, then it holds that $|x^b - y^b| \leq 2^{1-b}|x - y|^b$.

Lemma 2 [32, Lemma 2.4]: Let c and d be positive real numbers and $\gamma(x, y) > 0$ be a real-valued function. Then, it holds that $|x|^c|y|^d \leq (c\gamma(x, y)|x|^{c+d}/(c + d) + (d\gamma^{-c/d}(x, y)|y|^{c+d})/(c + d)$.

Appendix B
Proof of Proposition 2

Substituting (19) into (1) yields the closed-loop system as

$$\dot{p}_i = v_i$$
$$\dot{v}_i = \dot{\hat{v}}_i - k_2\left(\left(v_i - \hat{v}_i\right)^{1/\alpha} + k_1^{1/\alpha}\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left(p_i - p_j\right)\right)^{2\alpha-1}. \tag{38}$$

Consider the Lyapunov function candidate $W = (1/2)p_f^T p_f + (1/2)v_f^T v_f$. Taking the time derivative of W along the trajectory of system (38) yields

$$\dot{W} = p_f^T v_f + v_f^T u_f \leq W + \sum_{i=1}^{n_f}\|v_i\|\|u_i\| \tag{39}$$

where $u_f = [u_1^T, \ldots, u_{n_f}^T]$. It follows from Lemma 1 that:

$$\left\|y^b\right\| = \left(\sum_{m=1}^{d}\left(y_m^b\right)^2\right)^{1/2} \leq \sum_{m=1}^{d}|y_m|^b \leq d\left(\sum_{m=1}^{d} y_m^2\right)^{b/2} = d\|y\|^b.$$

It follows that: $\|u_i\| \leq \|\dot{\hat{v}}_i\| + d^{2\alpha}k_2(\|v_i\|^{2-1/\alpha} + \|\hat{v}_i\|^{2-1/\alpha}) + dk_1^{2-1/\alpha}k_2\|\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(p_i - p_j)\|^{2\alpha-1}$. Note that

$$\left\|\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}(p_i - p_j)\right\| \leq \sum_{i=1}^{n}\left(\|I_d\| + \|g_{ij}^*\|^2\right)\left(\|p_i\| + \|p_j\|\right) \leq 2n\|p_i\| + 2\sum_{j=1}^{n}\|p_j\|.$$

Then, we have

$$\|u_i\| \leq \left\|\dot{\hat{v}}_i\right\| + d^{2\alpha}k_2\left(\|v_i\|^{2-1/\alpha} + \left\|\hat{v}_i\right\|^{2-1/\alpha}\right) + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\left(n\|p_i\| + \sum_{j=1}^{n}\|p_j\|\right)^{2\alpha-1}.$$

Under Assumption 2, $\|\dot{v}_j(t)\|$ is bounded for all $t \geq t_0$ and $j \in \mathcal{V}_l$. Accordingly, for $t \in [t_0, t_0 + T_0]$, if Assumption 2 is satisfied, $\|v_j(t)\|$ and $\|p_j(t)\|$ are also bounded for all $t \geq t_0$ and $j \in \mathcal{V}_l$. Moreover, for any initial estimates $\hat{v}_i(t_0)$, which are well defined, the trajectories of $\hat{v}_i(t)$ are bounded, since the estimator (5) makes $\hat{v}_i(t)$ finite-time convergent. As a result, by the computation of the estimator (5), the trajectories of $\dot{\hat{v}}_i(t)$ are also bounded given the bounded trajectories of $\hat{v}_i(t)$. Therefore, there exists a constant ϑ_1 such that

$$\vartheta_1 \geq \left\|\dot{\hat{v}}_i\right\| + d^{2\alpha}k_2\left\|\hat{v}_i\right\|^{2-1/\alpha} + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\sum_{j=n_f+1}^{n}\left\|p_j\right\|^{2\alpha-1}$$

and it follows that:

$$\|u_i\| \leq \vartheta_1 + d^{2\alpha}k_2\|v_i\|^{2-1/\alpha} + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2 \times\left(n^{2\alpha-1}\|p_i\|^{2\alpha-1} + \sum_{j=1}^{n_f}\left\|p_j\right\|^{2\alpha-1}\right). \tag{40}$$

Substituting (40) into (39) yields

$$\begin{aligned}\dot{W} &\leq W + \vartheta_1\sum_{i=1}^{n_f}\|v_i\| + d^{2\alpha}k_2\sum_{i=1}^{n_f}\|v_i\|^{3-1/\alpha} \\ &\quad + (2n)^{2\alpha-1}dk_1^{2-1/\alpha}k_2\sum_{i=1}^{n_f}\|v_i\|\|p_i\|^{2\alpha-1} \\ &\quad + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\sum_{i=1}^{n_f}\sum_{j=1}^{n_f}\|v_i\|\left\|p_j\right\|^{2\alpha-1} \\ &\leq W + \vartheta_1\sum_{i=1}^{n_f}\|v_i\| + d^{2\alpha}k_2\sum_{i=1}^{n_f}\|v_i\|^{3-1/\alpha}\end{aligned}$$

$$+\frac{2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\left(n^{2\alpha-1}+n_f\right)}{2\alpha}\sum_{i=1}^{n_f}\|p_i\|^{2\alpha}$$
$$+\frac{2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\left(n^{2\alpha-1}+n_f\right)}{2\alpha}\sum_{i=1}^{n_f}\|v_i\|^{2\alpha}$$

where the last inequality holds by Lemma 2. By $\max(\|p_i\|^b, \|v_i\|^b) \leq (\|p_i\|^2 + \|v_i\|^2)^{b/2}$ for all $b \geq 0$, we have

$$\begin{aligned}\dot{W} \leq\; & W + \vartheta_1 \sum_{i=1}^{n_f}\left(\|p_i\|^2 + \|v_i\|^2\right)^{1/2} \\ & + d^{2\alpha}k_2 \sum_{i=1}^{n_f}\left(\|p_i\|^2 + \|v_i\|^2\right)^{(3-1/\alpha)/2} \\ & + \frac{2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\left(n^{2\alpha-1}+n_f\right)}{2\alpha}\sum_{i=1}^{n_f}\left(\|p_i\|^2 + \|v_i\|^2\right)^{\alpha}.\end{aligned}$$

By Lemma 1, we have $\sum_{i=1}^{n_f}(\|p_i\|^2+\|v_i\|^2)^{1/2} \leq \sum_{i=1}^{n_f}(\|p_i\|+\|v_i\|)$. Moreover, based on the equivalence between any two different kinds of norms in $\mathbb{R}^d$, there exists a constant $\vartheta_2 > 0$ such that

$$\sum_{i=1}^{n_f}(\|p_i\| + \|v_i\|) \leq \vartheta_2\left(\sum_{i=1}^{n_f}\left(\|p_i\|^2 + \|v_i\|^2\right)\right)^{1/2}.$$

It follows that $\sum_{i=1}^{n_f}(\|p_i\|^2 + \|v_i\|^2)^{1/2} \leq \vartheta_2 W^{1/2}$. Similarly, there exist positive constants $\vartheta_3 > 0$ and $\vartheta_4 > 0$ such that $\sum_{i=1}^{n_f}(\|p_i\|^2 + \|v_i\|^2)^{(3-1/\alpha)/2} \leq \vartheta_3 W^{(3-1/\alpha)/2}$ and $\sum_{i=1}^{n_f}(\|p_i\|^2 + \|v_i\|^2)^{\alpha} \leq \vartheta_4 W^{\alpha}$. Thus, we obtain

$$\begin{aligned}\dot{W} \leq\; & W + \vartheta_1\vartheta_2 W^{1/2} + d^{2\alpha}k_2\vartheta_3 W^{(3-1/\alpha)/2} \\ & + \frac{2^{2\alpha-1}dk_1^{2-1/\alpha}k_2\left(n^{2\alpha-1}+n_f\right)\vartheta_4}{2\alpha}W^{\alpha}.\end{aligned}$$

It follows from Lemma 2 that $W^b \leq bW + 1 - b,\ 0 < b \leq 1$. Then, we have

$$\dot{W} \leq \vartheta_5 W + \vartheta_6 \tag{41}$$

where $\vartheta_5 = 1 + (\vartheta_1\vartheta_2)/2 + d^{2\alpha}k_2\vartheta_3(3\alpha - 1)/2\alpha + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2(n^{2\alpha-1} + n_f)\vartheta_4(1 - \alpha)/(2\alpha)$ and $\vartheta_6 = (\vartheta_1\vartheta_2)/2 + d^{2\alpha}k_2\vartheta_3(1 - \alpha)/2\alpha + 2^{2\alpha-1}dk_1^{2-1/\alpha}k_2(n^{2\alpha-1} + n_f)\vartheta_4(1 - \alpha)/\alpha$. By direct integration on (41), W is bounded for $t \in [t_0, t_0 + T_0]$. Therefore, $p_f(t)$ and $v_f(t)$ are bounded for $t \in [t_0, t_0 + T_0]$. The proof is thus completed. ∎

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