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Adaptive Bearing-Only Control of Multiple Euler-Lagrange Systems for Static Geometric Formation ⋆

Bangwei Zhao * **Jianing Zhao** ** **Weiyao Lan** * **Xiao Yu** *,[1]

* *Department of Automation, Xiamen University, Xiamen, China (e-mail: bangweizhao@xmu.edu.cn; xiaoyu@xmu.edu.cn; wylan@xmu.edu.cn).*

** *Department of Automation, Shanghai Jiao Tong University, Shanghai, China (e-mail: jnzhao@sjtu.edu.cn)*

Abstract: This paper studies the formation control problem of multi-agent systems each of which the dynamics are modeled by the Euler-Lagrange equation with unknown parameters. The objective of each agent is to develop a control law using the bearing information with respect to its neighbors, such that the multi-agent system forms a static geometric formation defined by the bearing constraints among agents. An adaptive dynamic control law with a leader-following type is proposed such that the objective can be achieved in the presence of unknown systems' parameters. Finally, simulation examples are presented to illustrate the effectiveness and feasibility of the main results.

Keywords: Euler-Lagrange Systems, Bearing Control, Formation Control

1. INTRODUCTION

Formation control of multi-agent systems attracted much research attention, due to its potential application to the manipulation of networked robotics systems. According to the variety of target formations, a number of formation control methods have been proposed, among which different sensory feedback, for instance, displacement, distance, and bearings, could be used (Oh et al., 2015).

Plenty of existing formation control methods of multi-agent systems usually model the agents as a point described as single-integrators, double-integrators, or other simple systems. However, when it comes to the formation control of practical robotic systems, their dynamics cannot be neglected. For those robotic systems described by the Euler-Lagrange systems equation with uncertain parameters, the formation control problems become more challenging. Lu et al. (2019) proposed an adaptive tracking control law for an Euler-Lagrange system with disturbance rejection. In Ren (2009), a distributed model-independent consensus algorithm for networked Euler–Lagrange systems was processed to achieve leaderless formation consensus. Wang and Huang (2019) raised an adaptive control method for leader-following multi-agent systems with Euler-Lagrange dynamics and uncertain parameters. By adaptive control, the agents are able to track a sinusoidal signal. Besides, based on the backstepping method and graph rigidity theory, an adaptive formation controller was designed in Cai and de Queiroz (2015); Cai and Huang (2016). In Lu and Liu (2017), the leader-following consensus problem was investigated in the case with communication delays and switching networks. In Lu and Liu (2019a), the leader-following attitude consensus problem under switching networks was studied. In Lu and Liu (2019b), the leader-following consensus problem was resolved in the case with unknown dynamic leader.

The aforementioned formation control methods for either Euler-Lagrange systems or other systems such as Liu and Jiang (2013), Oh and Ahn (2011) and Wang et al. (2020) usually employ the sensory feedback from the position, distance, or displacement, which were comprehensively reviewed in Oh et al. (2015). Nevertheless, by these methods, agents need to obtain their position accurately, or they need to measure the relative distance or displacement with respect to their neighbors. Consequently, agents usually need more sophisticated sensors to perceive the environment and more equipment to communicate their states with each other. All these extra requirements could make the robotic formation more costly and thus limit the scope of application.

As a matter of fact, using bearings as the distributed feedback in formation control provides a new channel for the establishment of a practical multi-agent system. Different from distance and position, bearing measurement drops the distance measurement and can be easily obtained by a single camera using object recognition algorithms. With the development of bearing rigidity theory by Michieletto et al. (2016); Zelazo et al. (2014), especially Zhao and Zelazo (2016a,b), it has been proven that the shape of a topology network can only be determined by bearing information if the bearing constraints satisfy certain con-

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[1] Corresponding author.

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ditions. These researches provide theoretical fundamentals for the feasibility of formation control based on bearing information. Zhao et al. (2019); Zhao and Zelazo (2019) also developed some bearing-based or bearing-only formation control methods on single-integrators, double-integrators, or unicycles. Note that in these works, more than one leader is needed, and the followers move with the formation by measuring the bearings among them and their neighbors. Recently, to solve formation problems for multiple Euler-Lagrange systems by bearing measurements, an adaptive bearing-only control law was proposed such that the leaderless formation of the multi-agent system with uncertain parameters can be achieved.

This paper aims to solve the leader-following formation control problem of a multi-agent system with Euler-Lagrange dynamics containing unknown parameters. A graph is used to describe the network topology among the leaders and followers. An adaptive bearing-only formation control method for those followers is proposed to achieve the target formation. Finally, a physical experiment on mobile robots is presented to illustrate the effectiveness and feasibility.

The main contribution of this paper is twofold. First, compared with Zhao et al. (2019) where the bearing-only formation control problem was solved for multi-agent systems of double-integrators, our proposed approach resolves the problem for multiple Euler-Lagrange systems with unknown parameters which is more general in the aspect of system dynamics. Second, though the leaderless control law proposed in Li et al. (2021) could be used to solve the leader-following problem of Euler-Lagrange systems, our proposed control law is more simple in the sense of computing the time-varying matrix in the controller, which appears more feasible for online implementation and is also shown by the physical experiment on mobile robots.

The rest of the paper is organized as follows. Section II presents the notations, preliminaries, and problem formulation. Section III shows our main result. Simulation and experimental results are given in Section IV. Finally, conclusions are drawn in Section VII.

2. NOTATIONS AND PROBLEM FORMULATION

2.1 Euler-Lagrange systems

Consider n agents in $\mathbb{R}^d$ $(d \geq 2)$, where $p_i \in \mathbb{R}^d$ and $q_i \in \mathbb{R}^d$ denote the position of the center point of mass and the head point of the i-th agent, respectively. Take $d = 3$ as an example and mark $p_i = [x_{ci},\ y_{ci},\ z_{ci}]^{\mathrm{T}}$, $q_i = [x_i,\ y_i,\ z_i]^{\mathrm{T}}$. Then, we have

$$q_i = p_i + L_i \begin{bmatrix} \sin\beta_i \cos\alpha_i \\ \sin\beta_i \sin\alpha_i \\ \cos\beta_i \end{bmatrix} \tag{1}$$

where L_i is the distance between q_i and p_i, and $[L_i,\ \alpha_i,\ \beta_i]^{\mathrm{T}}$ is the spherical coordinates of q_i with p_i as the origin, α_i as the azimuthal angle and β_i as the polar angle. The corresponding diagram can be found in Fig.1. Note that in Fig. 1, axes x', y', z' are parallel with the axes X, Y, Z in the global coordinate frame respectively.

By defining $\eta_i = [\mu_i,\ \omega_{\alpha_i},\ \omega_{\beta_i}]^{\mathrm{T}}$ where μ_i and $\omega_{\alpha_i}, \omega_{\beta_i}$ is the linear velocity and the angular velocity of the i-th

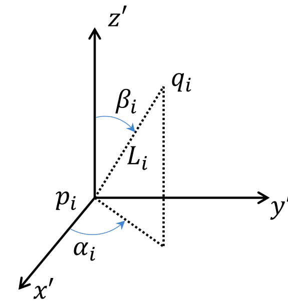

Fig. 1. Schematic diagram of L_i, α_i, β_i

robot, the dynamics of those agents are described as

$$\bar{M}_i \dot{\eta}_i + \bar{D}_i \eta_i = u_i \tag{2}$$

where $\bar{M}_i = \mathrm{diag}(m_i,\ I_{1_i},\ I_{2_i})$ with m_i and I_{1_i}, I_{2_i} are the mass and inertia of the i-th agent and $\bar{D}_i \in \mathbb{R}^{3\times 3}$ is a constant damping matrix. Then, write

$$\eta_i = J_i \dot{q}_i \tag{3}$$

with

$$J_i = \begin{bmatrix} s\beta_i c\alpha_i & s\beta_i s\alpha_i & c\beta_i \\ -s\alpha_i / L_i s\beta_i & c\alpha_i / L_i s\beta_i & 0 \\ -c\beta_i c\alpha_i / L_i & -c\beta_i s\alpha_i / L_i & s\beta_i / L_i \end{bmatrix} \tag{4}$$

where $s\alpha$ and $c\alpha$ denote $\sin\alpha$ and $\cos\alpha$ respectively.

Through some mathematical transformations, the Euler-Lagrange equation can be obtained as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + D_i(q_i)\dot{q}_i = \tau_i \tag{5}$$

where $M_i(q_i)$ is the inertia matrix, $C_i(q_i, \dot{q}_i)$ is the Coriolis and centripetal matrix, and $D_i(q_i)$ is the gravity matrix. These matrices can be calculated as

$$\tau_i = J_i^{\mathrm{T}} \bar{u}_i$$

$$M_i = J_i^{\mathrm{T}} \bar{M}_i J_i,\ C_i = J_i^{\mathrm{T}} \bar{M}_i \dot{J}_i,\ D_i = J_i^{\mathrm{T}} \bar{D}_i J_i$$

It is well-known that the Euler-Lagrange system (5) has the following three important properties which are important for controller designing.

Property 1: $M_i(q_i)$ is a symmetric positive matrix.

Property 2: For all $\kappa_i \in \mathbb{R}^n$,

$$\begin{aligned} M_i(q_i)\dot{\kappa}_i + C_i(q_i, \dot{q}_i)\kappa_i + D_i(q_i)\dot{q}_i \\ = Y_i(q_i, \dot{q}_i, \kappa_i, \dot{\kappa}_i)\theta_i \end{aligned} \tag{6}$$

where $\theta_i \in \mathbb{R}^r$ is a constant vector that contains a series of physical parameters of the robot with r representing the dimension of the parameter vector and $Y_i(q_i, \dot{q}_i, \kappa_i, \dot{\kappa}_i) \in \mathbb{R}^{d\times r}$ is a known matrix subject to the state of the agents and could be extracted by matrix operations.

Property 3: For all $q_i, \dot{q}_i \in \mathbb{R}^d$, $\left(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)\right)$ is a skew-symmetric matrix.

For simplicity, we denote $M_i(q_i)$ as M_i, $C_i(q_i, \dot{q}_i)$ as C_i and $D_i(q_i)$ as D_i respectively in the rest of the paper.

2.2 Bearing and bearing rigidity theory

Let

$$e_{ij} = q_j - q_i, \quad g_{ij} = e_{ij} / \|e_{ij}\| \tag{7}$$

be the edge and bearing vector respectively, where $\|\cdot\|$ denotes Euclidean norm of a vector.

Consider a multi-agent system that contains n agents of which the dynamics are modeled by (5). We use a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to describe the interaction among the agents, where $\mathcal{V}$ denotes the set of the n agents and $\mathcal{E}$ denotes the set of the m edges. Agent i is said to be able to measure the bearing information with respect to an agent j if an edge $(i, j) \in \mathcal{E}$. Besides, let $\mathcal{N}_i$ be the set of all agents that can be measured by agent i, i.e., the neighbor of agent i. Accordingly, we use

$$e = [e_1^{\mathrm{T}} \dots e_m^{\mathrm{T}}] \in \mathbb{R}^{dm},\ g = [g_1^{\mathrm{T}} \dots g_m^{\mathrm{T}}] \in \mathbb{R}^{dm}$$

to denote the compact vector of all edges and bearings in a graph $\mathcal{G}$ respectively.

Moreover, in order to describe the graph more explicitly, we define a matrix H as follows: $[H]_{ik} = 1$ if node i is the head of the edge k; $[H]_{ik} = -1$ if node i is the tail of edge k; and $[H]_{ik} = 0$ otherwise. We denote $\bar{H} = H \otimes \mathbf{1}_n$, where $\otimes$ denotes their Kronecker product and $\mathbf{1}_n$ is an n-dimension identity matrix.

Before presenting the problem formulation, we introduce the concept of bearing rigidity as follows. From Zhao and Zelazo (2016a), the bearing rigidity matrix of a graph $\mathcal{G}$ is a matrix defined as

$$R = \frac{\partial g}{\partial q} \in \mathbb{R}^{dm \times dn} \tag{8}$$

According to Zhao and Zelazo (2016a), the bearing rigidity matrix can also be described as

$$R(p) = \mathrm{diag}\left(\frac{P_{g_k}}{\|e_k\|}\right) \bar{H} \tag{9}$$

where

$$P_{g_k} = I_d - \frac{g_k}{\|g_k\|} \frac{g_k^{\mathrm{T}}}{\|g_k\|} \tag{10}$$

is an orthogonal projection matrix and satisfies

$$P_{g_k} g_k = 0. \tag{11}$$

The following lemma from Zhao et al. (2019) introduces the property of the so-called bearing rigidity.

Lemma 1. For a directed graph $\mathcal{G}$ which contains n agents defined in $\mathbb{R}^d$, the graph $\mathcal{G}$ is said to be infinitesimal bearing rigid if $R\,(p)$ satisfies

$$\mathrm{rank}\,(R\,(p)) = dn - d - 1. \tag{12}$$

For a directed graph that satisfies infinitesimal bearing rigidity, there is no node that could move alone without changing any bearing in the graph, which indicates that the shape of the graph is unique.

In particular, for a leader-following formation, define the bearing-based graph Laplacian matrix as

$$[\mathcal{B}]_{ij} = \begin{bmatrix} \mathcal{B}_{ll} & \mathcal{B}_{lf} \\ \mathcal{B}_{fl} & \mathcal{B}_{ff} \end{bmatrix} = \begin{cases} 0_{d \times d}, & i \neq j, (i,j) \notin \mathcal{E}, \\ -P_{g_{ij}^*}, & i \neq j, (i,j) \in \mathcal{E}, \\ \sum\limits_{k \in \mathcal{N}_i} P_{g_{ij}^*}, & i = j, i \in \mathcal{V}. \end{cases} \tag{13}$$

According to (Zhao et al., 2019, Lemma 1), the position of followers q_i^* could be uniquely determined by the bearings $g_{ij}^*, (i,j) \in \mathcal{E}$ and leaders' motion $q_k, k \in \mathcal{V}_l$ if and only if the sub-matrix $\mathcal{B}_{ff} \in \mathbb{R}^{dn_f \times dn_f}$ is non-singular.

2.3 Problem Formulation

Consider a leader-following multi-agent formation system denoted by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which contains $l (l \geq 2)$ leaders and n followers and m edges. The formation of agents is denoted by $\mathcal{G}(q)$, $q = [q_1^{\mathrm{T}} \dots q_n^{\mathrm{T}}]$, where node i of the graph $\mathcal{G}$is mapped to q_i. The target formation is defined as follows.

Definition 1. (target formation). The formation of $\mathcal{G}(q)$ is said to be a target formation $\mathcal{G}(q^*)$ if all bearings satisfy

$$g_{ij} = g_{ij}^*, i, j \in \mathcal{E}. \tag{14}$$

Thus, it holds that $p_i = p_i^*$ and $v_i = v_i^* = \mathbf{0}$ for the target formation.

Now, the considered problem is formulated as follows.

Problem 1: Consider n agents containing n followers to be controlled. and l static leaders and the network topology is described by a directed graph $\mathcal{G}$. Given a target formation $\mathcal{G}(q^*)$ defined by target bearing g_{ij}^*, design the torque of each follower using the bearings with respect to its neighbor as feedback, such that the target formation $\mathcal{G}(q^*)$ can be achieved.

In other words, the objective is to design τ_i to ensure $g_{ij} \to g_{ij}^*$, and then $v_i \to v_i^* = \mathbf{0}$ and $p_i \to p_i^*$ according to bearing-rigidity theory. To solve this problem, the following assumptions are needed.

Assumption 1: All agents will not collide with each other remaining a certain distance from each other.

Assumption 2: All agents could measure the bearings they need during the whole motion.

Assumption 3: The target formation is unique, that is, the bearing rigidity matrix of the digraph $\mathcal{G}$ satisfies (12) and the corresponding Bearing Laplacian Matrix is non-singular.

Remark 2.1. Assumptions 1 and 2 guarantee that the bearings(along with their 1st and 2nd order deviation) are always measurable and bounded, which results that the formation control law could be executed, while assumption 3 provides the uniqueness of the shape of the desired formation. These assumptions are common, and similar assumptions are made in bearing-based control, such as Trinh et al. (2019), Li et al. (2022), and so on.

3. MAIN RESULTS

Before illustrating our main result which solves Problem 1, by marking v_i as $\dot{q}_i$ we first transform the expressions of Euler-Lagrange systems 5 to the form as

$$\begin{cases} \dot{q}_i = v_i, \\ \dot{v}_i = M_i^{-1} \tau_i - M_i^{-1} \left(C_i + D_i\right) v_i. \end{cases} \tag{15}$$

System (15) can be viewed as a second-order system and thus we can handle the acceleration $\dot{v}_i$ of each agent by properly designing the control torques τ_i.

Combining (15) with the second property of the Euler-Lagrange equation, and suppose that parameter vector θ_i is known, we can use the right term of the equation $Y(v_i, \dot{v}_i, \kappa_i, \dot{\kappa}_i)\theta_i$ which is viewed to be known as part of the system input. Since κ_i in (6) could be arbitrarily chosen, let κ_i be the speed of the leaders $\mu \in \mathcal{R}^d$ which is set to $\mathbf{0}$ when the leaders are static.

However, θ_i may be a vector containing unknown or uncertain variables in practice. Thus, we propose an adaptive controller with an estimation of these unknown variables as

$$\begin{cases} \tau_i = Y(q_i, \dot{q}_i, 0, 0)\,\hat{\theta}_i + k_p \sum_{j\in\mathcal{N}_i} \left(g_{ij} - g_{ij}^*\right) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij} \\ \dot{\hat{\theta}}_i = -\Lambda_i Y(q_i, \dot{q}_i, 0, 0)^{\mathrm{T}} v_i \end{cases} \tag{16}$$

where k_p and k_v are positive constant parameters, and $\Lambda \in \mathbb{R}^{r\times r}$ is a positive definite matrix gain to handle the convergence rate of the estimator. Note that we use the velocity error v_i and the known matrix $Y(q_i, \dot{q}_i, 0, 0)$ to perform estimation for the unknown parameter θ_i.

Remark 3.1. It should be pointed out that though we have used q and $\dot{q}$ in the expression of Y in the control law (16), instead of containing parameters beyond bearings and the posture of the agents themselves, as in (4), yet there are only some angles (α_i, β_i) and system states in the expression of this matrix.

Remark 3.2. In the control law (16), the computation on the matrix $Y(q_i, \dot{q}_i, 0, 0)$ is used. While the control law in Li et al. (2021) requires the computation on the matrix $Y(q_i, \dot{q}_i, v_s, \dot{v}_s)$ with $v_s = kR(q)g^*$, where $k > 0$ and $R(q)$ is the bearing-rigidity matrix. Thus, the control law (16) appears more friendly to online computation and real-time implementation.

Now, we present our main result as follows.

Theorem 1. Under Assumptions 1–3, the state q of the multi-agent system (15) under the controller (16) converges to q^* when the leaders are static.

Proof: For simplicity, we denote the bearing part as

$$\dot{v}_{s_i} = k_p \sum_{j\in\mathcal{N}_i} \left(g_{ij} - g_{ij}^*\right) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij}. \tag{17}$$

Substituting the controller (16) into the system dynamics (15), we have

$$\begin{cases} \dot{q}_i = v_i, \\ \dot{v}_i = M_i^{-1}\dot{v}_{s_i} - M_i^{-1}(C_i + D_i)\,v_i + \\ \qquad M_i^{-1} Y(q_i, \dot{q}_i, 0, 0)\,\hat{\theta}_i \\ \dot{\hat{\theta}}_i = -\Lambda_i Y(q_i, \dot{q}_i, 0, 0)^{\mathrm{T}} v_i. \end{cases} \tag{18}$$

Denote

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i$$

as the estimate error of the parameters. Then, according to Property 2 of Euler-Lagrange systems, we have

$$\begin{aligned} Y(q_i, \dot{q}_i, 0, 0)\,\hat{\theta}_i &= Y(q_i, \dot{q}_i, 0, 0)\,(\theta_i - \tilde{\theta}_i) \\ &= -Y(q_i, \dot{q}_i, 0, 0)\,\tilde{\theta}_i + D_i v_i. \end{aligned} \tag{19}$$

By defining position error and speed error as

$$\delta_{q_i} = q_i - q_i^*, \ \delta_{v_i} = v_i$$

respectively, where q_i^* is the target position of the head of agent i at instant t, we have $\dot{q}_i^* = 0$. Then, we obtain the following error dynamics

$$\begin{cases} \dot{\delta}_{q_i} = \delta_{v_i} \\ \dot{\delta}_{v_i} = M_i^{-1}\dot{v}_{s_i} - M_i^{-1}C_i\delta_{v_i} - M_i^{-1}Y(q_i, \dot{q}_i, 0, 0)\,\tilde{\theta}_i \\ \dot{\tilde{\theta}}_i = -\Lambda_i Y(q_i, \dot{q}_i, 0, 0) \end{cases} \tag{20}$$

Now, consider the following Lyapunov candidate

$$V = k_p e^{\mathrm{T}}(g - g^*) + \frac{1}{2}\delta_v^{\mathrm{T}} M \delta_v + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Lambda^{-1}\tilde{\theta} \tag{21}$$

where $e = \left[e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, ..., e_k^{\mathrm{T}}\right]^{\mathrm{T}}$, $g = [g_1^{\mathrm{T}}, g_2^{\mathrm{T}}, ..., g_k^{\mathrm{T}}]^{\mathrm{T}}$, $\delta_v = [\delta_{v_1}^{\mathrm{T}}, \delta_{v_2}^{\mathrm{T}}, ..., \delta_{v_n}^{\mathrm{T}}]^{\mathrm{T}}$, $M = \mathrm{blkdiag}(M_1(q_1),\ M_n(q_n),\ ...,\ M_n(q_n))$, $\tilde{\theta} = [\tilde{\theta}_1^{\mathrm{T}},\ ..., \tilde{\theta}_k^{\mathrm{T}}]^{\mathrm{T}}$. Let $C = \mathrm{blkdiag}\,(C_1, C_2,\ ..., C_n)$, $q = [q_{L_1}^{\mathrm{T}}, ..., q_{L_l}^{\mathrm{T}}, q_1^{\mathrm{T}}, ..., q_n^{\mathrm{T}}]^{\mathrm{T}}$ and $v = \dot{q}$.

From the definition of bearing, we know that for any two bearings g_i and g_j, their inner product $g_i^{\mathrm{T}} g_j$ reach its maxima if and only if $g_i = g_j$ since they are unit vectors. Thus, it follows that

$$e^{\mathrm{T}}(g - g^*) = \|e\| g^{\mathrm{T}}(g - g^*) \geq 0 \tag{22}$$

and $e^{\mathrm{T}}(g - g^*) = 0$ if and only if $g = g^*$, which means that the system locates at the target point by Lemma 1. Note that both M and Λ are positive definite matrices, the Lyapunov candidate (21) is positive definite and radially unbounded.

Before calculating the time-derivative of the Lyapunov candidate (21), we present the following relations (Zhao et al., 2019),

$$\dot{e_k} = \dot{e_{ij}} = \dot{q}_j - \dot{q}_i \tag{23}$$

$$\dot{e} = \bar{H}\dot{q} \tag{24}$$

and

$$\dot{g_k} = P_{g_k}\dot{e}_k / \|e_k\| \tag{25}$$

$$e^{\mathrm{T}}\dot{g} = \Sigma_k e_k^{\mathrm{T}} P_{g_k}\dot{e}_k / \|e_k\| = 0. \tag{26}$$

Then, the time-derivative of the Lyapunov candidate along the trajectories of system (20) is

$$\begin{aligned} \dot{V} &= k_p e^{\mathrm{T}}\dot{g} + k_p (g - g^*)^{\mathrm{T}} \bar{H}\dot{q} + \frac{1}{2}\delta_v^{\mathrm{T}}\dot{M}\delta_v + \delta_v^{\mathrm{T}} M\dot{\delta}_v \\ &\quad - \tilde{\theta}^{\mathrm{T}}\Lambda^{-1}\dot{\hat{\theta}} \\ &= k_p (g - g^*)^{\mathrm{T}} \bar{H} v - \tilde{\theta}^{\mathrm{T}}\Lambda^{-1}\left\{\Lambda Y^{\mathrm{T}}\delta_v + \dot{\hat{\theta}}\right\} \\ &\quad + \frac{1}{2}\delta_v^{\mathrm{T}}\left\{\dot{M}\delta_v + 2M\left(M^{-1}\dot{v}_s - M^{-1}C\delta_v\right)\right\} \\ &= k_p (g - g^*)^{\mathrm{T}} \bar{H} v + \frac{1}{2}\delta_v^{\mathrm{T}}\left\{\left(\dot{M} - 2C\right)\delta_v + 2\dot{v}_s\right\}. \end{aligned} \tag{27}$$

According to Property 3 of Euler-Lagrange systems, we have $\delta_v^{\mathrm{T}}\left(\dot{M} - 2C\right)\delta_v = 0$, then (27) can be translated to

$$\begin{aligned} \dot{V} &= k_p (g - g^*)^{\mathrm{T}} \bar{H} v + \delta_v^{\mathrm{T}}\dot{v}_s \\ &= k_p (g - g^*)^{\mathrm{T}} \bar{H} v - k_p\delta_v^{\mathrm{T}}\bar{H}^{\mathrm{T}}(g - g^*) - k_v\delta_v^{\mathrm{T}}\bar{H}^{\mathrm{T}}\dot{g} \\ &= k_p (g - g^*)^{\mathrm{T}} \bar{H} v - k_p (g - g^*)^{\mathrm{T}} \bar{H} v - k_v v^{\mathrm{T}}\bar{H}^{\mathrm{T}}\dot{g} \\ &= -k_v v^{\mathrm{T}}\bar{H}^{\mathrm{T}}\dot{g}. \end{aligned} \tag{28}$$

Then, according to the conclusions in Zhao et al. (2019), since $\bar{H}q = e$, $\bar{H}v = \bar{H}\dot{q} = \dot{e}$, then we have:

$$\dot{V} = -k_v\dot{e}^{\mathrm{T}}\dot{g} = -k_v\sum_{k=1}^{m}\dot{e}_k^{\mathrm{T}}\dot{g}_k = -k_v\sum_{k=1}^{m}\dot{e}_k^{\mathrm{T}}\frac{P_{g_k}}{\|e_k\|}\dot{e}_k \leq 0. \tag{29}$$

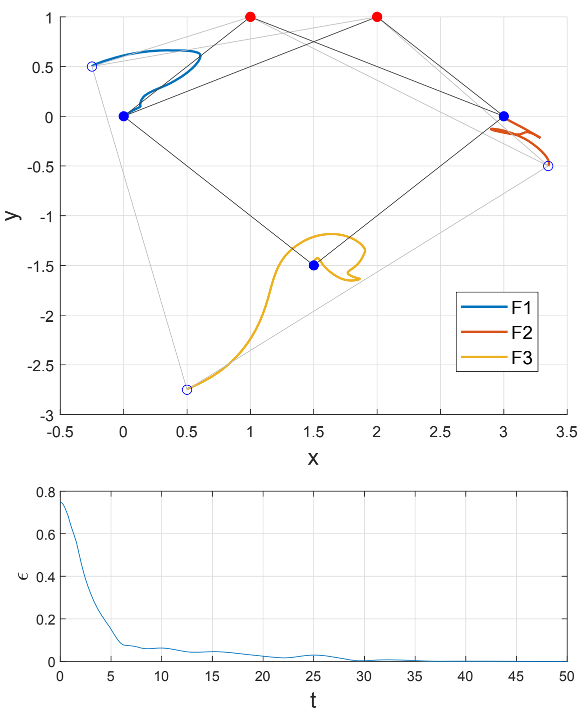

Fig. 2. Trajectories of followers in 2-D and curve of bearing error ϵ in the motion with $k_v = 15,\ k_p = 10$

Finally, V is non-increasing and bounded in t, which implies that $\lim_{t\to\infty}\int_0^t \dot{V}dt$ exists and is finite. It follows from (28) that $\ddot{V} = -k_v\dot{v}^{\mathrm{T}}\bar{H}^{\mathrm{T}}\dot{g} - k_v v^{\mathrm{T}}\bar{H}^{\mathrm{T}}\ddot{g}$ is also bounded based on Assumption 1. This results in $\lim_{t\to\infty}\dot{V} = 0$ by Barbalet's lemma.

Let $D = \{\delta_q \in \mathcal{R}^{nd},\ \delta_v \in \mathcal{R}^{nd},\ \hat{\theta} \in \mathcal{R}^{np}\}$, V is positive definite in D and $\dot{V}$ is negative semi-definite. Then, it is required to find $S = \{\delta_q, \delta_v, \hat{\theta} \in D|\dot{V} = 0\}$. Since

$$\dot{V} = 0 \Rightarrow \delta_v = \mathbf{0},$$

we have $S = \{\delta_q, \delta_v, \hat{\theta} \in D|\dot{V} = 0\}$. Let $\{\delta_q(t), \delta_v(t), \hat{\theta}(t)\}$ be a solution that belongs identically to S, we have $\delta_v \equiv \mathbf{0} \Rightarrow \dot{\delta_v} \equiv \mathbf{0},\ v_i \equiv \mu,\ \dot{g} \equiv \mathbf{0} \Rightarrow \Sigma_{j\in\mathcal{N}_i}\delta_{g_{ij}} \equiv Y_i\tilde{\theta}_i$. Note that $Y(q_i, v_i, 0, 0)$ will become $\mathbf{0}$, and then we have $\Sigma_{j\in\mathcal{N}_i}\delta_{g_{ij}} \equiv 0$. Finally, according to the bearing rigidity theory, we have $q_i \equiv q_i^*$ which means all the agents have reached the target position and Problem 1 is solved. This completes the proof.

Remark 3.3. The proposed algorithm cannot ensure the asymptotic stability when the leaders begin to move, since those followers still cannot precisely estimate the unknown parameters. The estimated residual significantly affects the controller's performance, leaving a bounded bearing-defined tracking error. A more efficient parameter estimator may drive the system to the target position which is one of the future topics.

4. SIMULATION EXAMPLES

In this section, we present two numerical examples to illustrate the effectiveness and feasibility of the proposed approach. There are five agents which are defined in $\mathbb{R}^2$ in the first case while the second case contains eight underwater-like robots.

In order to evaluate the formation performance, define the tracking error as $\epsilon = \|g - g^*\|$ with the current bearing and target bearing. Obviously, it holds that $\epsilon = 0$ if and only if the followers are all located at the desired position.

Firstly, a simple 2-D example with 2 leaders and 3 followers is given. The simulation result could be found in Fig. 2 with the trajectories and the corresponding bearing error. In this and the subsequent trajectory figure, we mark the initial position of the agents as hollow 'O's and the corresponding final position as solid 'O's. Meanwhile, the red solid 'O's represent the leaders and the blue solid ones represent the followers. Besides, those gray lines represent the initial formation relationship and the black ones represent the final scene.

The system parameters are set to $L_i = 0.15, M_i = \mathrm{diag}(1.3, 0.4)$ and $D_i = \mathrm{diag}(0.3, 0.004)$ and the initial states of the followers are set as

$$\begin{aligned} q_1(0) =&[-0.25,\ 0.5]^{\mathrm{T}}, q_2(0) = [3.35,\ -0.5]^{\mathrm{T}},\\ q_3(0) =&[0.5,\ -2.75]^{\mathrm{T}},\\ v_i(0) =&[0.1,\ 0.2]^{\mathrm{T}}, \alpha_i(0) = 0, \hat{\theta}_i(0) = \mathbf{0}. \end{aligned}$$

The second example of 3-D agents is shown in Fig. 3. In this example, we set a cubic-like target formation where there exist two leaders (red) and six followers (blue), as described in Fig. 3. Each follower has at least three neighbors which link them through the black lines and could measure the corresponding bearings with respect to them. Meanwhile, those leaders keep static as anchors for the target formation.

All the parameters are set to those as in Li et al. (2021) with $L_i = 0.15$, $\bar{M}_i = \mathrm{diag}(1.3, 0.4, 0.5)$ and $\bar{D} = \mathrm{diag}(0.3, 0.004, 0.5)$. The initial position is omitted here while the initial states are set to

$$v_i(0) = [0, 0, 0]^{\mathrm{T}},\ \alpha_i(0) = 0,\ \beta_i(0) = 0,\ \hat{\theta}_i(0) = \mathrm{zeros}(p),\\ \forall i = 1, 2, ..., 6.$$

Fig. 3 shows the trajectories of the followers represented by colored solid lines. The evolution of bearing error ϵ is also shown in Fig. 3 while Fig. 4 shows some other states of each follower including the linear velocities u_i and angular velocities $\omega_{\alpha_i},\ \omega_{\beta_i}$.

The target formations are achieved in these two simulation results with the bearing error decreasing to zero. This shows that the proposed control law is efficient for the formation control of Euler-Lagrange-like models.

5. CONCLUSION

We present a bearing-only formation control method for followers with Euler-Lagrange dynamics containing unknown parameters in this paper. A static leader-following geometric formation can be achieved by the proposed adaptive control law. We also show its feasibility through numerical experiments to illustrate the practicability. In the future, the global asymptotically stable control law for the closed-loop system under moving leaders will be analyzed, and estimating leaders' states from bearings will also be investigated.

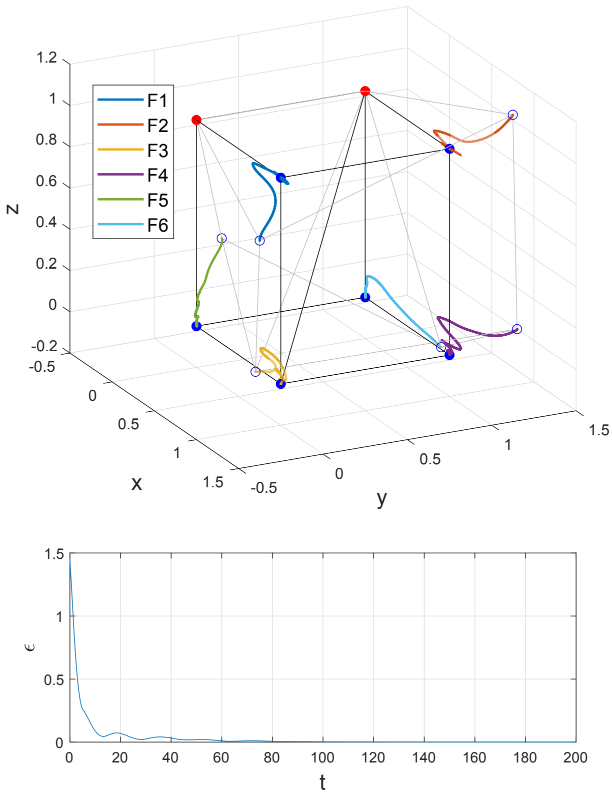

Fig. 3. Trajectories of 3-D followers and curve of bearing error ϵ in the motion with $k_v = 25,\ k_p = 10$

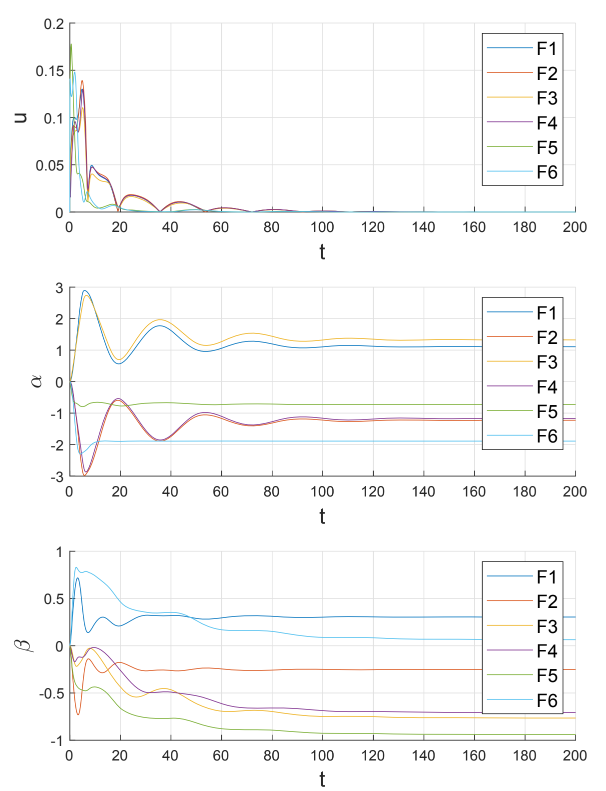

Fig. 4. Curve of the corresponding linear and angular velocity in the simulation

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